04/29/08

Space Shuttle - 38 thrusters

- has different configurations: one is gravity-gradient mode, used to minimize noise from thrusters during microgravity experiments.

$$M = -3n^2 \begin{bmatrix} (J_2 - J_3)\theta_1 \\ -(J_3 - J_1)\theta_2 \\ 0 \end{bmatrix}$$

Linearized EOM for s/c in LEO (alt ~ 500 km):

$$J_1 \ddot{\theta}_1 - n(J_1 - J_2 + J_3)\dot{\theta}_3 + 4n^2(J_2 - J_3)\theta_1 = 0$$
$$J_2 \ddot{\theta}_2 - 3n^2(J_3 - J_1)\theta_2 = 0$$
$$J_3 \ddot{\theta}_3 + n(J_1 - J_2 + J_3)\dot{\theta}_1 + n^2(J_2 - J_1)\theta_3 = 0$$

Recall,

$a_1 = \frac{J_2 - J_3}{J_1}$, $a_3 = \frac{J_2 - J_1}{J_3}$. Look at just roll + yaw:

$$\ddot{\theta}_1 + n(a_1 - 1)\dot{\theta}_3 + 4n^2 a_1 \theta_1 = 0$$
$$\ddot{\theta}_3 + n(1 - a_3)\dot{\theta}_1 + n^2 a_3 \theta_3 = 0$$

① Laplace

$$(s^2 + 4n^2 a_1)\hat{\theta}_1 + sn(a_1 - 1)\hat{\theta}_3 = 0$$
$$sn(1 - a_3)\hat{\theta}_1 + (s^2 + n^2 a_3)\hat{\theta}_3 = 0$$

② Solve + find char. eq

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_3 \end{bmatrix} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$\begin{bmatrix} \theta_1 \\ \theta_3 \end{bmatrix} = \frac{1}{bc - ad} \begin{bmatrix} \text{(scribbled out)} \end{bmatrix} \begin{bmatrix} -a & c \\ b & -d \end{bmatrix} \begin{bmatrix} u_1 \\ u_3 \end{bmatrix}$$

$$s^4 + (1 + 3a_1 + a_1 a_3)n^2 s^2 + 4a_1 a_3 n^4 = 0$$

$w = s^2$

$$w = \frac{-(1+3a_1+a_1a_3)n^2 \pm \sqrt{(1+3a_1+a_1a_3)^2 n^4 - 16a_1a_3n^4}}{2}$$

So,

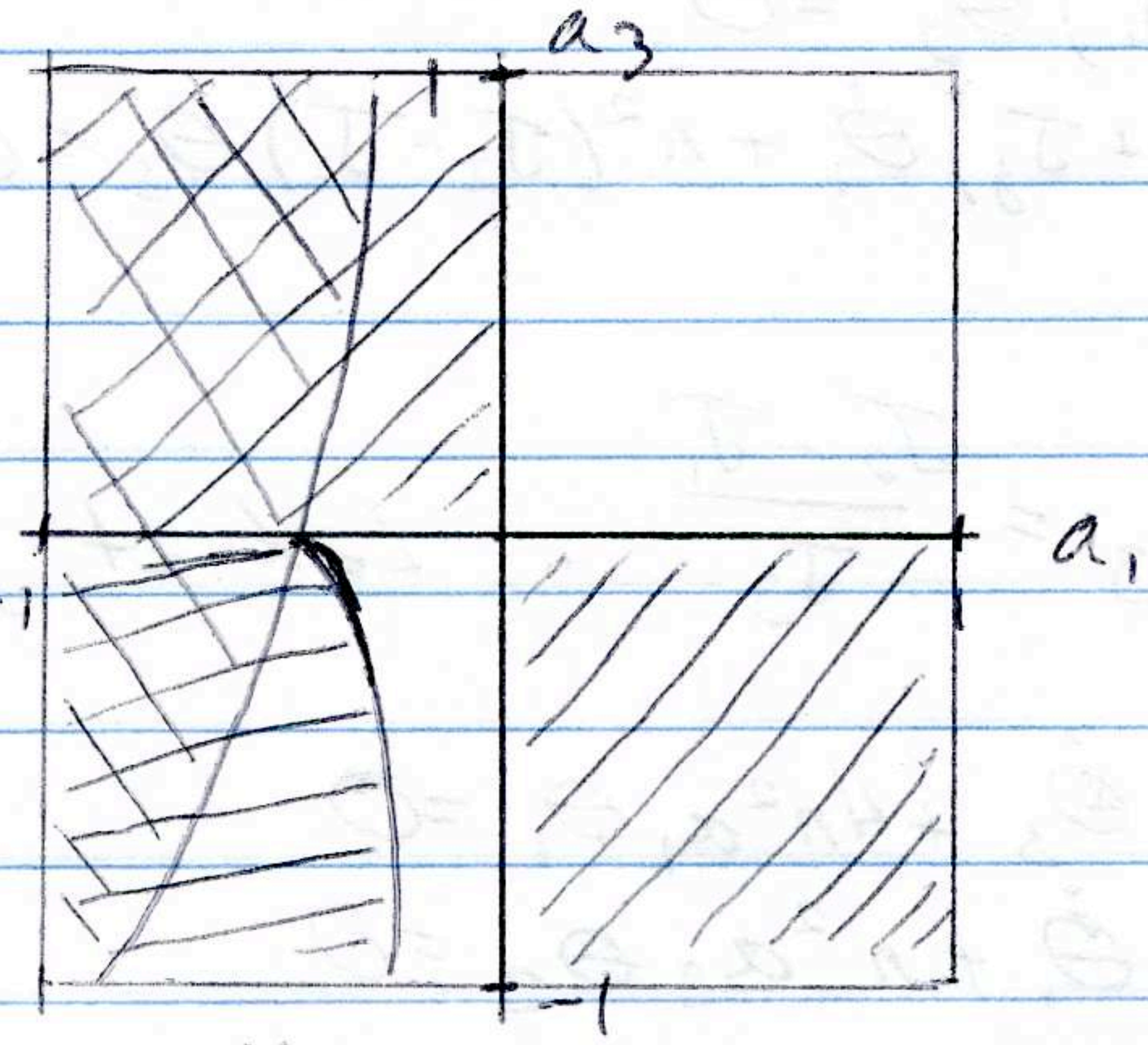
$s = \pm\sqrt{w}$. It is always true that $Re(\sqrt{a+bi}) \geq 0$ which means the best we can hope for is neutral stability occuring when $w < 0$.

Conditions:

$$(1+3a_1+a_1a_3)^2 - 16a_1a_3 > 0$$

$$1+3a_1+a_1a_3 > 0$$

$$a_1a_3 > 0$$



Now looking at the pitch eqn,

$$s^2 + 3n^2\left(\frac{J_1 - J_3}{J_2}\right) = 0$$

Then, for stability:

$$J_1 > J_3.$$

To relate this to a statement about our "a"s, let's go back to our **J** matrix.

$$J = \begin{bmatrix} J_1 & 0 & 0 \\ 0 & J_2 & 0 \\ 0 & 0 & J_3 \end{bmatrix}.$$ By their definitions,

$J_1 = \int (r_2^2 + r_3^2)\, dm$

$J_2 = \int (r_1^2 + r_3^2)\, dm$

$J_3 = \int (r_1^2 + r_2^2)\, dm$

It is trivial, then to see that,

$$J_1 + J_2 = \int (r_1^2 + r_2^2)\,dm + 2\int r_3^2\,dm > J_3 \text{. Likewise,}$$

$$\boxed{\begin{aligned} J_1 + J_2 &> J_3 \\ J_2 + J_3 &> J_1 \\ J_3 + J_1 &> J_2 \end{aligned}}$$

. Therefore, $|a_1| = \left| \frac{J_2 - J_3}{J_1} \right| < 1$ and $|a_3| < 1$.

$$a_1 - a_3 = \left(\frac{J_2 - J_3}{J_1} \right) - \left(\frac{J_2 - J_1}{J_3} \right)$$

$$= \frac{J_3(J_2 - J_3) - J_1(J_2 - J_1)}{J_1 J_3}$$

$$= \frac{J_2 J_3 - J_3^2 - J_1 J_2 + J_1^2}{J_1 J_3} = \left[(J_3 - J_1) J_2 + (J_1^2 - J_3^2) \right] / J_1 J_3$$

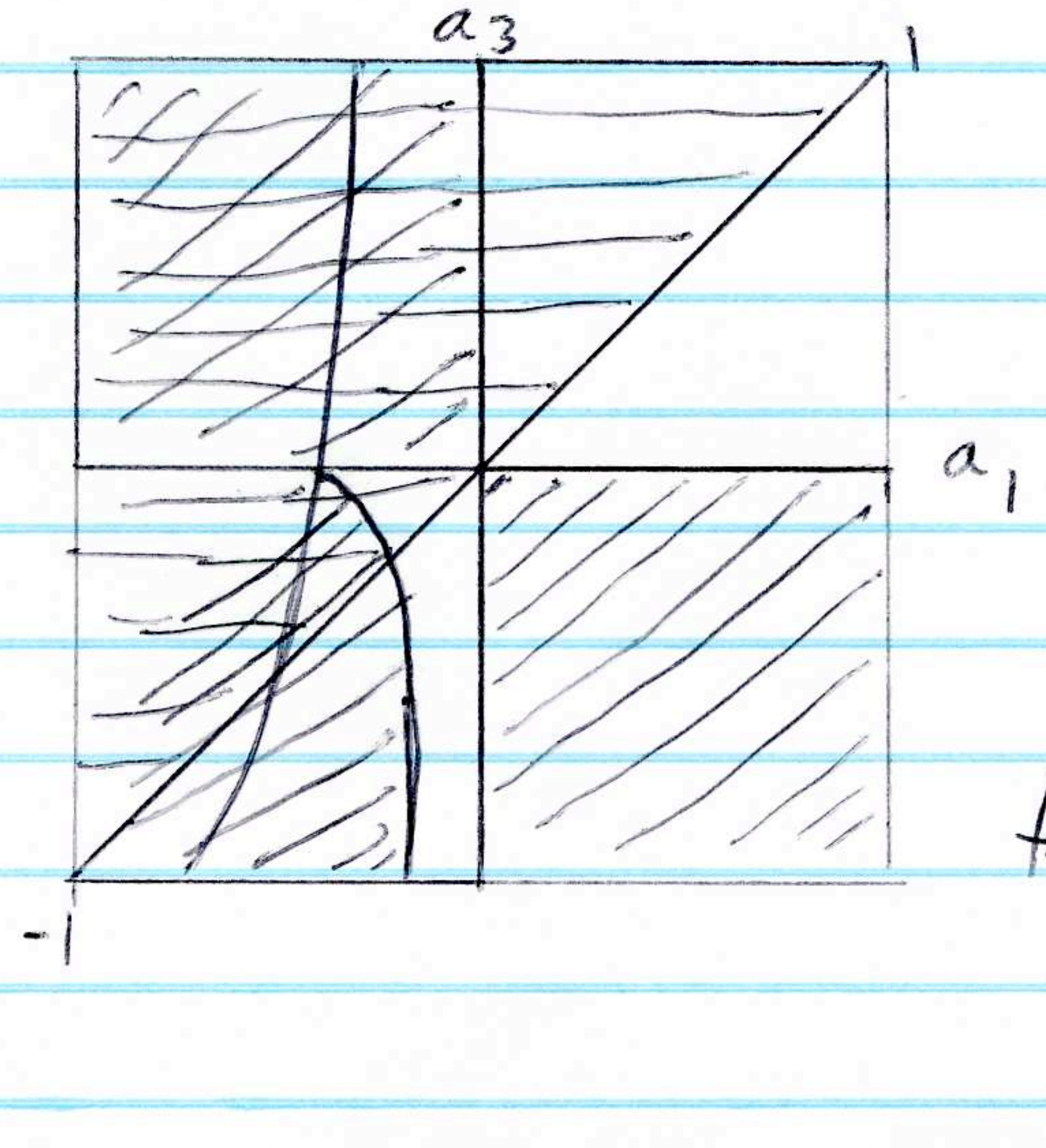
$$= \left[(J_1 + J_3)(J_1 - J_3) - J_2 (J_1 - J_3) \right] / J_1 J_3$$

$$= \Big[\underbrace{\big((J_1 + J_3) - J_2\big)}_{\text{pos. by physical constraint}} \underbrace{(J_1 - J_3)}_{\text{req. pos. for stability}} \Big] / J_1 J_3 = a_1 - a_3$$

This means,

$$a_1 > a_3 \,!$$

Now our domain looks like



for neutral gravity-gradient stability.