

$$\dot{\Theta} = A^{-1}(w + nb) \Rightarrow \dot{w} = J^{-1}(M + u - s(w)Jw), \quad \dot{h} = -u - s(w)h$$

$$a_1 = \frac{J_2 - J_3}{J_1}, \quad a_3 = \frac{J_2 - J_1}{J_3} \Rightarrow a_1 - 1 = \frac{J_1 - J_2 + J_3}{J_1}, \quad 1 - a_3 = \frac{J_1 - J_2 + J_3}{J_3}$$

$$\Rightarrow (M_1 + u_1)/J_1 = \ddot{\Theta}_1 + n(a_1 - 1)\dot{\Theta}_3 + n^2 a_1 \Theta_1 \quad \text{and apply Laplace}$$

$$\Rightarrow (M_3 + u_3)/J_3 = \ddot{\Theta}_3 + n(1 - a_3)\dot{\Theta}_1 + n^2 a_3 \Theta_3$$

$$\xrightarrow{T_1} \frac{M_1 + u_1}{J_1} = (s^2 + n^2 a_1)\Theta_1 + sn(a_1 - 1)\Theta_3 \quad \frac{M_3 + u_3}{J_3} = sn(1 - a_3)\Theta_1 + (s^2 + n^2 a_3)\Theta_3 = T_3$$

$$(s^2 + n^2 a_1)(s^2 + n^2 a_3) - s^2 n^2 (a_1 - 1)(1 - a_3) = (s^2 + n^2)(s^2 + a_1 a_3 n^2) \quad \text{char. eqn. for open loop}$$

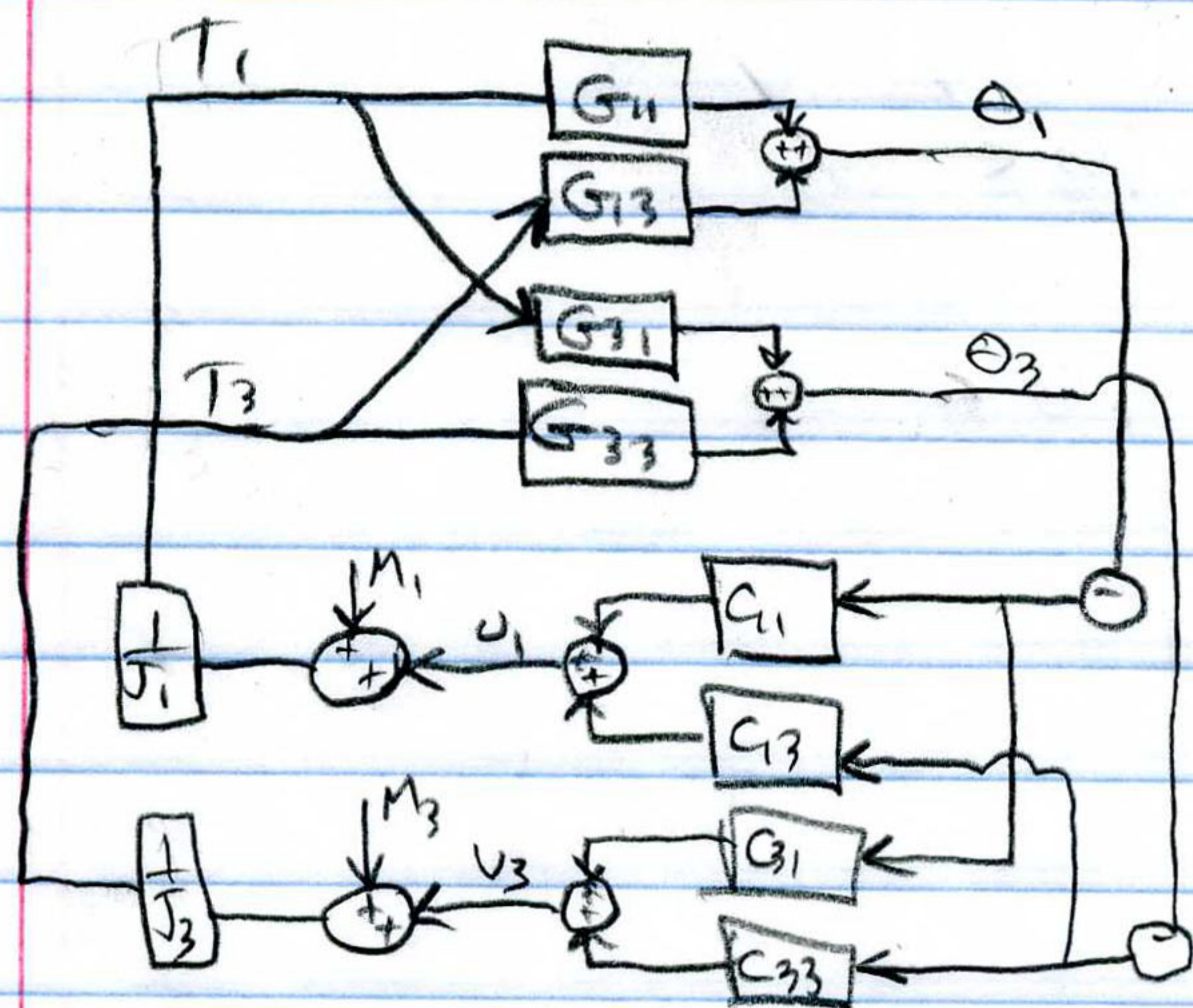
$$\Rightarrow \Theta_3 = \frac{T_3 - sn(1 - a_3)\Theta_1}{s^2 + n^2 a_3} \Rightarrow T_1 = (s^2 + n^2 a_1)\Theta_1 + sn(a_1 - 1) \left( \frac{T_3 - sn(1 - a_3)\Theta_1}{s^2 + n^2 a_3} \right) \Theta_1$$

$$\Rightarrow T_1 (s^2 + n^2 a_3) - T_3 sn(a_1 - 1) = \Theta_1 [(s^2 + n^2 a_1)(s^2 + n^2 a_3) - s^2 n^2 (a_1 - 1)(1 - a_3)]$$

$$D = (s^2 + n^2)(s^2 + a_1 a_3 n^2)$$

$$\Rightarrow \Theta_1 = \frac{s^2 + n^2 a_3}{D} T_1 + \frac{-sn(a_1 - 1)}{D} T_3 = G_{11} T_1 + G_{13} T_3 = \frac{N_{11}}{D} T_1 + \frac{N_{13}}{D} T_3$$

$$\Rightarrow \Theta_3 = \frac{-sn(1 - a_3)}{D} T_1 + \frac{(s^2 + n^2 a_1)}{D} T_3 = G_{31} T_1 + G_{33} T_3 = \frac{N_{31}}{D} T_1 + \frac{N_{33}}{D} T_3$$



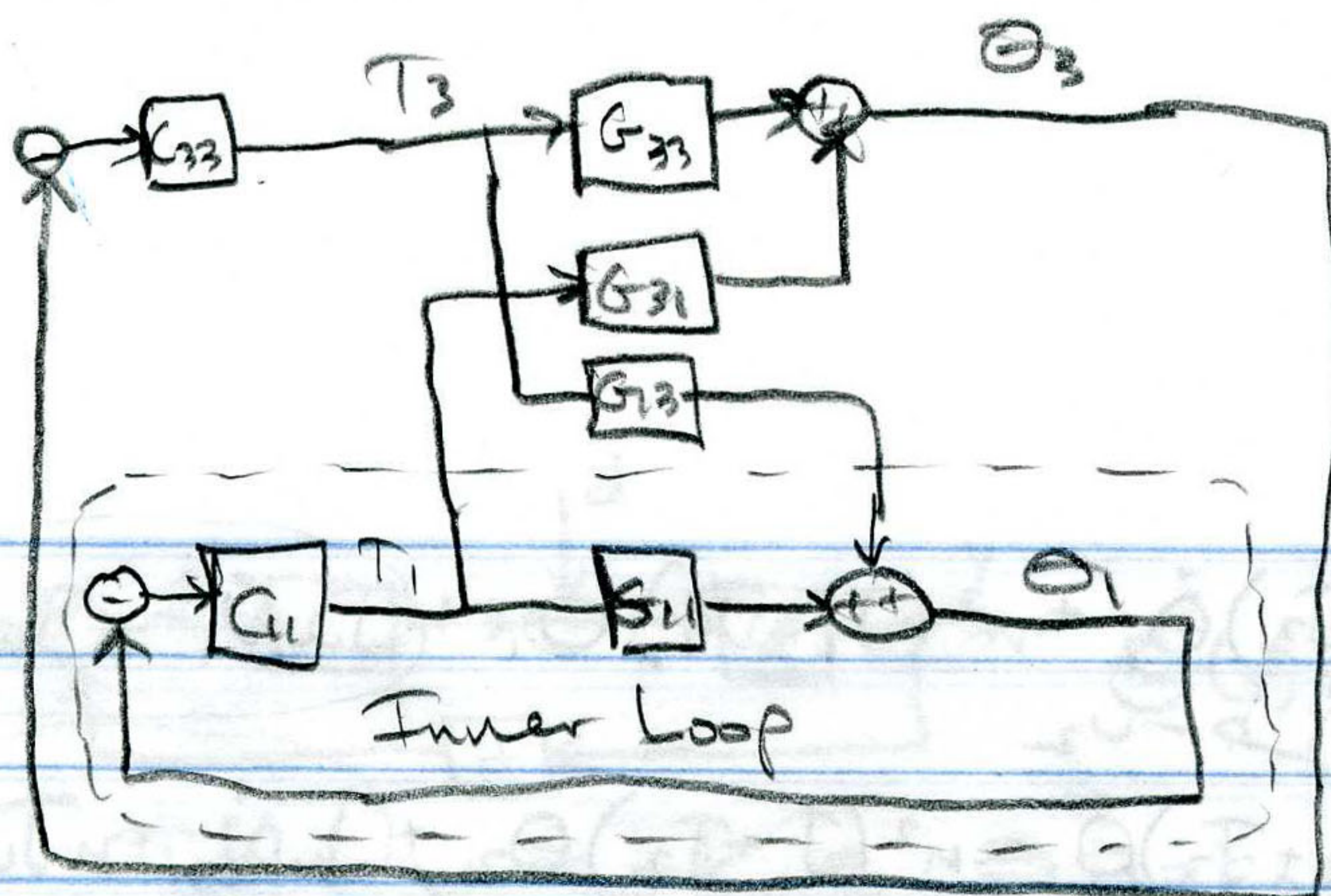
To do this by hand, kill  $G_{13} + C_{31}$ .

$$\begin{bmatrix} \Theta_1 \\ \Theta_3 \end{bmatrix} = \begin{bmatrix} G_{11} & G_{13} \\ G_{31} & G_{33} \end{bmatrix} \begin{bmatrix} T_1 \\ T_3 \end{bmatrix}$$

$$\begin{bmatrix} U_1 \\ U_3 \end{bmatrix} = \begin{bmatrix} C_{11} & \cancel{C_{13}} \\ \cancel{C_{31}} & C_{33} \end{bmatrix} \begin{bmatrix} \Theta_1 \\ \Theta_3 \end{bmatrix} = \begin{bmatrix} C_{11} & 0 \\ 0 & C_{33} \end{bmatrix} \begin{bmatrix} \Theta_1 \\ \Theta_3 \end{bmatrix}$$

then consider only loop that goes through  $\Theta_1$ .

So erase  $C_{13}$  and  $C_{31}$  and redraw the block diagram.



Stabilize inner loop first then outer loop.

Called successive loop closure