THIS RESULT LEAVES US WITH

$$M_1 = J_1\dot{\omega}_1 + J_w\dot{\nu}_1 - (J_2 - J_3)\omega_2\omega_3 + J_w(\omega_2\nu_3 - \omega_3\nu_2)$$
$$M_2 = J_2\dot{\omega}_2 + J_w\dot{\nu}_2 - (J_3 - J_1)\omega_3\omega_1 + J_w(\omega_3\nu_1 - \omega_1\nu_3)$$
$$M_3 = J_3\dot{\omega}_3 + J_w\dot{\nu}_3 - (J_1 - J_2)\omega_1\omega_2 + J_w(\omega_1\nu_2 - \omega_2\nu_1)$$

PLUGGING IN OUR NEW ω VALUES, & NEGLECTING SMALL PRODUCTS

$$\Rightarrow M_1 = J_1(\ddot{\theta}_1 - n\dot{\theta}_3) + J_w\dot{\nu}_1 - (J_2 - J_3)(-n\dot{\theta}_3 - n^2\theta_1) + J_w(-n\nu_3)$$
$$M_2 = J_2\ddot{\theta}_2 + J_w\dot{\nu}_2 \leftarrow -u_2$$
$$M_3 = J_3(\ddot{\theta}_3 + n\dot{\theta}_1) + J_w\dot{\nu}_3 - (J_1 - J_2)(-n\dot{\theta}_1 + n^2\theta_3) + J_w(n\nu_1)$$

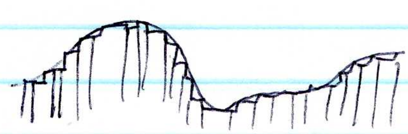
SIMPLIFY...

$$M_1 = J_1\ddot{\theta}_1 - n(J_1 - J_2 + J_3)\dot{\theta}_3 + n^2(J_2 - J_3)\theta_1 + J_w\dot{\nu}_1 - nJ_w\nu_3$$
$$M_2 = \sim$$
$$M_3 = J_3\ddot{\theta}_3 + n(J_1 - J_2 + J_3)\dot{\theta}_1 - n^2(J_1 - J_2)\theta_3 + J_w\dot{\nu}_3 + nJ_w\nu_1$$

SAT O' THE DAY: (1996) LANDS ON SOLAR ARRAYS ON AN ASTEROID & SENDS BACK IMAGES. EROS WAS THE ASTEROID NEAR SPACECRAFT; GOT THERE FEB. 01 — 4/10/08

800 kg w/ 300 PROPELLANT 3AXIS ACTIVE CONTROL. 4 RXN WHEELS; 0.1° ANGLE TOLERANCE

$$J\ddot{\theta} = u = -k_p\theta - k_d\dot{\theta}$$



KINEMATIC DIFF. EQNS

$$\rightarrow \omega = A\dot{\theta} - nb$$

e's ARE UNIT VECTORS

$$b = R_{123}e_2$$

$$A = [R_3R_2e_1 \quad R_3e_2 \quad e_3]$$

Dynamic Eq'ns

$$M_1 = J_1\dot{\omega}_1 - (J_2 - J_3)\omega_2\omega_3 + J_\omega\dot{\nu}_1 + J_\omega(\omega_2\nu_3 - \omega_3\nu_2) \Leftarrow u_1$$

$$M_2 = J_2\dot{\omega}_2 - (J_3 - J_1)\omega_3\omega_1 + J_\omega\dot{\nu}_2 + J_\omega(\omega_3\nu_1 - \omega_1\nu_3) \leftarrow -u_2$$

$$M_3 = J_3\dot{\omega}_3 - (J_1 - J_2)\omega_1\omega_2 + J_\omega\dot{\nu}_3 + J_\omega(\omega_1\nu_2 - \omega_2\nu_1) \leftarrow -u_3$$

$$-u_1 = J_\omega\dot{\nu}_1 + J_\omega(\omega_2\nu_3 - \omega_3\nu_2)$$

$$-u_2 = J_\omega\dot{\nu}_2 + J_\omega(\omega_3\nu_1 - \omega_1\nu_3)$$

$$-u_3 = J_\omega\dot{\nu}_3 + J_\omega(\omega_1\nu_2 - \omega_2\nu_1)$$

$$h = J_\omega\begin{bmatrix}\nu_1\\ \nu_2\\ \nu_3\end{bmatrix} \Rightarrow -u = \dot{h} + S(\omega)h$$

$$\boxed{\dot{h} = -u - S(\omega)h}$$

Now,

$$\boxed{\begin{aligned}\dot{\Theta} &= A^{-1}(\omega + nb)\\ \dot{\omega} &= J^{-1}(M + u - S(\omega)J\omega)\\ \dot{h} &= -u - S(\omega)h\end{aligned}}$$

$$dy/dt = f(t, y)$$

Look @ linearized model

$$M_1 + u_1 = J_1\ddot{\Theta}_1 - n(J_1 - J_2 + J_3)\dot{\Theta}_3 + n^2(J_2 - J_3)\Theta_1$$

$$M_2 + u_2 = J_2\ddot{\Theta}_2$$

$$M_3 + u_3 = J_3\ddot{\Theta}_3 + n(J_1 - J_2 + J_3)\dot{\Theta}_1 - n^2(J_1 - J_2)\Theta_3$$

$$\left.\begin{aligned}u_1 &= J_\omega\dot{\nu}_1 - nJ_\omega\nu_3\\ u_2 &= J_\omega\dot{\nu}_2\\ u_3 &= J_\omega\dot{\nu}_3 + nJ_\omega\nu_1\end{aligned}\right\}$$

Use these eq'ns when you want a controller that depends on spin rate like

$$u_1 = -k_p\Theta_1 - k_d\dot{\Theta}_1 - k_h h_1$$

TO UNCOUPLE DYNAMIC EQN'S, INTRODUCE

$$a_1 = \frac{J_2 - J_3}{J_1} \quad , \quad a_3 = \frac{J_2 - J_1}{J_3}$$

$$a_1 - 1 = -\frac{J_1 - J_2 + J_3}{J_1}$$

$$1 - a_3 = \frac{J_1 - J_2 + J_3}{J_3}$$

$$(M_1 + u_1)/J_1 = \ddot{\theta}_1 + n(a_1 - 1)\dot{\theta}_3 + n^2 a_1 \theta_1$$
$$(M_3 + u_3)/J_3 = \ddot{\theta}_3 + n(1 - a_3)\dot{\theta}_1 + n^2 a_3 \theta_3$$

$$\frac{M_1 + u_1}{J_1} = (s^2 + n^2 a_1)\theta_1 + sn(a_1 - 1)\theta_3$$

$$\frac{M_3 + u_3}{J_3} = sn(1 - a_3)\theta_1 + (s^2 + n^2 a_3)\theta_3$$