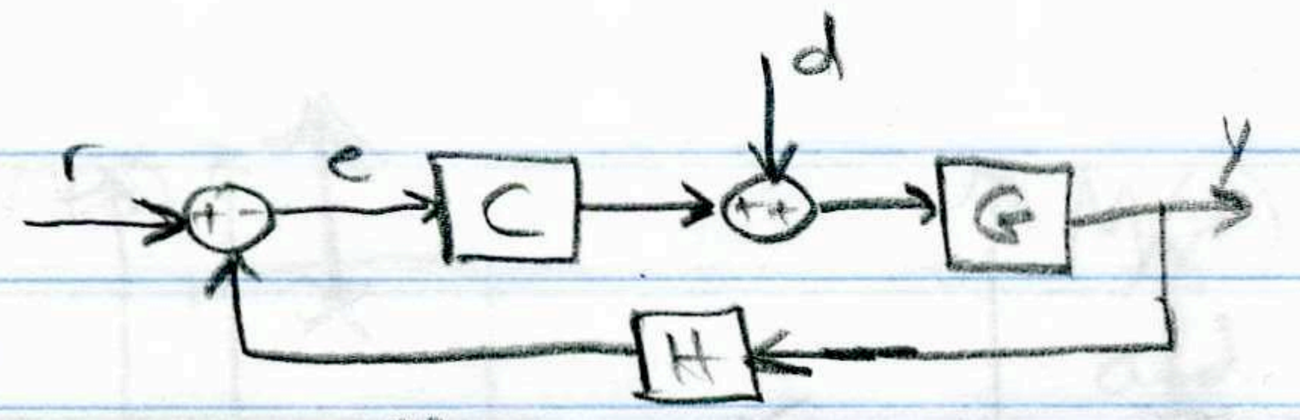


$$G(s)=\frac{\Theta(s)}{U(s)}=\frac{1}{Js^2},\quad C(s)=\frac{U(s)}{E(s)}=-k_p$$

$$J\ddot{\Theta}=u=+k_p e \Rightarrow u=k_p e+k_d\dot{e}$$

$$\Rightarrow U=(k_p+sk_d)E \text{ and } C=(k_p+sk_d)$$

ex: $$\begin{bmatrix}\omega_1\\\omega_2\\\omega_3\end{bmatrix}=\begin{bmatrix}c_2c_3 & s_3 & 0\\ -c_2s_3 & c_3 & 0\\ s_2 & 0 & 1\end{bmatrix}\begin{bmatrix}\dot{\Theta}_1\\\dot{\Theta}_2\\\dot{\Theta}_3\end{bmatrix}\approx\begin{bmatrix}1 & \Theta_3 & 0\\ -\Theta_3 & 1 & 0\\ \Theta_2 & 0 & 1\end{bmatrix}\begin{bmatrix}\dot{\Theta}_1\\\dot{\Theta}_2\\\dot{\Theta}_3\end{bmatrix}\approx\begin{bmatrix}\dot{\Theta}_1\\\dot{\Theta}_2\\\dot{\Theta}_3\end{bmatrix}$$

$\bar{H}=J_1\omega_1\hat{b}_1+J_2\omega_2\hat{b}_2+J_3\omega_3\hat{b}_3$ and add in reaction wheel

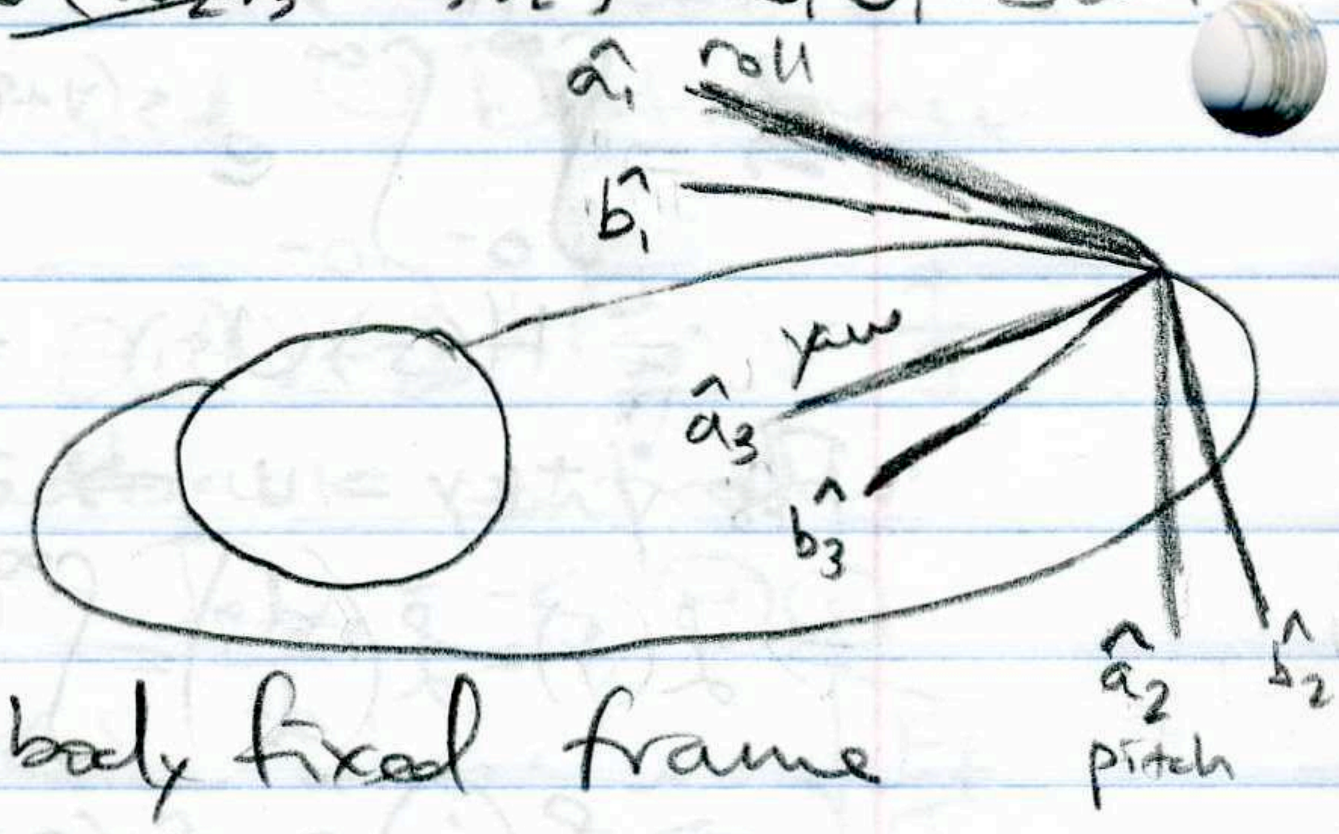
$$\Rightarrow \bar{H}=(J_1\omega_1+J_w\nu_1)\hat{b}_1+(J_2\omega_2+J_w\nu_2)\hat{b}_2+(J_3\omega_3+J_w\nu_3)\hat{b}_3$$

and $M=\dot{H}+S(\omega)H$ plug in values to get Euler's eqns

$$\Rightarrow\begin{bmatrix}M_1\\M_2\\M_3\end{bmatrix}=\begin{bmatrix}J_1\dot{\omega}_1+J_w\dot{\nu}_1\\J_2\dot{\omega}_2+J_w\dot{\nu}_2\\J_3\dot{\omega}_3+J_w\dot{\nu}_3\end{bmatrix}+\begin{bmatrix}0 & -\omega_3 & \omega_2\\ \omega_3 & 0 & -\omega_1\\ -\omega_2 & \omega_1 & 0\end{bmatrix}\begin{bmatrix}J_1\omega_1+J_w\nu_1\\J_2\omega_2+J_w\nu_2\\J_3\omega_3+J_w\nu_3\end{bmatrix}$$

$$\Rightarrow\begin{cases}M_1=J_1\dot{\omega}_1+J_w\dot{\nu}_1-\cancel{(J_2-J_3)\omega_2\omega_3}^{0}+J_w\cancel{(\omega_2\nu_3-\omega_3\nu_2)}^{0}=J_1\ddot{\Theta}_1+J_w\dot{\nu}_1\\ M_2=J_2\dot{\omega}_2+J_w\dot{\nu}_2=J_2\ddot{\Theta}_2+J_w\dot{\nu}_2\\ M_3=J_3\dot{\omega}_3+J_w\dot{\nu}_3=J_3\ddot{\Theta}_3+J_w\dot{\nu}_3\end{cases}$$

$$\Rightarrow\begin{cases}M_1+u_1=J_1\ddot{\Theta}_1\\ M_2+u_2=J_2\ddot{\Theta}_2\\ M_3+u_3=J_3\ddot{\Theta}_3\end{cases}\Rightarrow M_i+u_i=J_i\ddot{\Theta}_i$$



where N-inertial, A-orbit, and B-body fixed frame

$n=.011$ rad/s = orbital angular rate

$$\Rightarrow {}^N\bar{\omega}^B={}^N\bar{\omega}^A+{}^A\bar{\omega}^B=-n\hat{a}_2+{}^A\bar{\omega}^B$$

$$\Rightarrow\begin{bmatrix}\omega_1\\\omega_2\\\omega_3\end{bmatrix}=\begin{bmatrix}c_2c_3 & s_3 & 0\\ -c_2s_3 & c_3 & 0\\ s_2 & 0 & 1\end{bmatrix}\begin{bmatrix}\dot{\Theta}_1\\\dot{\Theta}_2\\\dot{\Theta}_3\end{bmatrix}-n\begin{bmatrix}c_3s_1s_2+c_1s_3\\c_1c_3-s_1s_2s_3\\-c_2s_1\end{bmatrix}$$ using 1-2-3 Euler angles

$$\Rightarrow\begin{bmatrix}\omega_1\\\omega_2\\\omega_3\end{bmatrix}=\begin{bmatrix}1 & \Theta_3 & 0\\ -\Theta_3 & 1 & 0\\ \Theta_2 & 0 & 1\end{bmatrix}\begin{bmatrix}\dot{\Theta}_1\\\dot{\Theta}_2\\\dot{\Theta}_3\end{bmatrix}-n\begin{bmatrix}\Theta_3\\1\\-\Theta_1\end{bmatrix}=\begin{bmatrix}\dot{\Theta}_1-n\Theta_3\\\dot{\Theta}_2-n\\\dot{\Theta}_3+n\Theta_1\end{bmatrix}$$

$$\Rightarrow\begin{cases}M_1=J_1\dot{\omega}_1+J_w\dot{\nu}_1-(J_2-J_3)\omega_2\omega_3+J_w(\omega_2\nu_3-\omega_3\nu_2)\\ M_2=J_2\dot{\omega}_2+J_w\dot{\nu}_2-(J_3-J_1)\omega_3\omega_1+J_w(\omega_3\nu_1-\omega_1\nu_3)\\ M_3=J_3\dot{\omega}_3+J_w\dot{\nu}_3-(J_1-J_2)\omega_1\omega_2+J_w(\omega_1\nu_2-\omega_2\nu_1)\end{cases}$$

$$\Rightarrow\begin{cases}M_1=J_1(\ddot{\Theta}_1-n\dot{\Theta}_3)+J_w\dot{\nu}_1-(J_2-J_3)(-n\dot{\Theta}_3-n^2\Theta_1)+J_w(-n\nu_3)\\ M_2=J_2\ddot{\Theta}_2+J_w\dot{\nu}_2\\ M_3=J_3(\ddot{\Theta}_3+n\dot{\Theta}_1)+J_w\dot{\nu}_3-(J_1-J_2)(-n\dot{\Theta}_1+n^2\Theta_3)+J_w(n\nu_1)\end{cases}$$

$$\Rightarrow \begin{cases} M_1 = J_1\ddot{\theta}_1 - n(J_1 - J_2 + J_3)\dot{\theta}_3 + n^2(J_2 - J_3)\theta_1 + \underbrace{J_w\dot{\nu}_1 - nJ_w\nu_3}_{-u_1} \\ M_2 = J_2\ddot{\theta}_2 + J_w\dot{\nu}_2 \\ M_3 = J_3\ddot{\theta}_3 + n(J_1 - J_2 + J_3)\dot{\theta}_1 - n^2(J_1 - J_2)\theta_3 + \underbrace{J_w\dot{\nu}_3 + nJ_w\nu_1}_{-u_3} \end{cases}$$