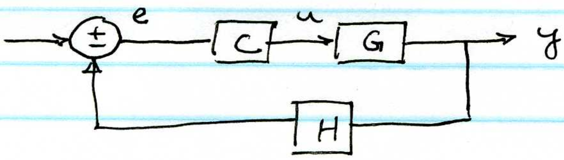


$$y = CGe = CG(r - Hy)$$

$$\Rightarrow \frac{y}{r} = \frac{CG}{1+\cancel{CG}\,CGH}$$

$$e = r - Hy = r - H(CGe)$$

$$\frac{e}{r} = \frac{1}{1+CGH}$$

## FINAL VALUE THEOREM

$$\lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s)$$

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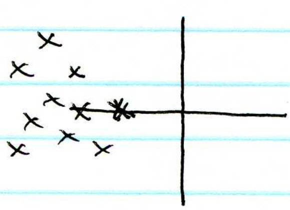
4/3/08

SOHO SATELLITE : SOLAR OBSERVER ; 1.5 MIL KM OUT FROM SUN @ $L_1$ ; RXN WHEELS & THRUSTERS

QUESTIONS FROM THE HW?

→ CRITICALLY DAMPED STUFF



$$\frac{N(s)}{s^2 + 2\xi\omega_n + \omega_n^2}$$

LET'S SAY WE HAVE A BUNCH OF POLES...

WE'D BE MAKING THE DENOMINATOR BIGGER

IT'D LOOK LIKE

$$\frac{N(s)}{D(s)} = \frac{\quad}{s+a} + \frac{\quad}{(s+a)^2} + \frac{\quad}{(s^2+2\xi\omega_n+\omega_n^2)}$$

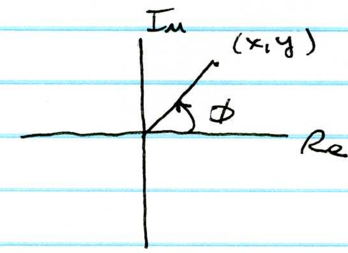
THE 3RD TERM GETS YOU

$$\ldots e^{-\zeta\omega_n t} \sin\left(\omega_n\sqrt{1-\zeta^2}\, t\right)$$

HOW ABOUT GAIN & PHASE?

$$|x + jy| = \sqrt{x^2+y^2}$$

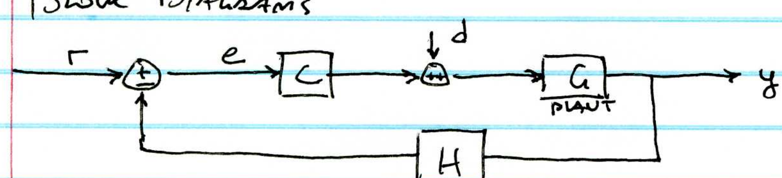
$$\angle(x+jy) = \tan^{-1}\left(\frac{y}{x}\right)$$




WHAT IF WE HAVE $\frac{a+jb}{c+jd}$ ?

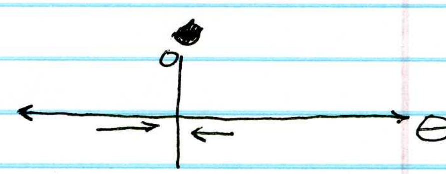
$$= \frac{(a+jb)(c-jd)}{(c+jd)(c-jd)} \Rightarrow \ldots \frac{(ac+bd)+j(bc-ad)}{c^2+d^2}$$

BLOCK DIAGRAMS




$$J\ddot{\theta} = u$$

$$G(s) = \frac{\Theta(s)}{U(s)} = \frac{1}{Js^2}$$




$u = + K_p e$

SIGN ERROR! THEY SHOULD ALL BE (+)

$$C(s) = \frac{U(s)}{E(s)} = + K_p$$

WHAT IF $u = +K_p e + K_d\dot{e}$ ? $\Rightarrow$ $C(s) = +(K_p + sK_d)$

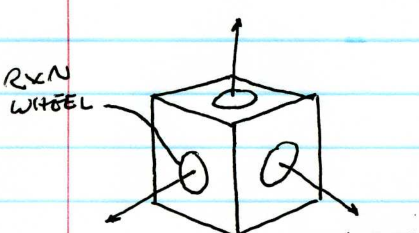
# CASE STUDY: 3-AXIS CONTROL

- 1-2-3 = ROLL - PITCH - YAW

$$\begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix} = \begin{bmatrix} c_2 c_3 & s_3 & 0 \\ -c_2 s_3 & c_3 & 0 \\ s_2 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix}$$ LINEARIZE...

$$\approx \begin{bmatrix} 1 & \theta_3 & 0 \\ -\theta_3 & 1 & 0 \\ \theta_2 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix}$$ ASSUME SMALL $\dot{\theta}$'s ...

$$\approx \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix}$$




$$\vec{H} = J_1 \omega_1 \hat{b}_1 + J_2 \omega_2 \hat{b}_2 + J_3 \omega_3 \hat{b}_3$$

ADD IN RXN WHEEL CONTRIBUTION

$$\hookrightarrow \vec{H} = (J_1 \omega_1 + J_\omega \nu_1)\hat{b}_1 + (J_2 \omega_2 + J_\omega \nu_2)\hat{b}_2 + (J_3 \omega_3 + J_\omega \nu_3)\hat{b}_3$$

$$M = \dot{H} + S(\omega)H \implies \begin{bmatrix} M_1 \\ M_2 \\ M_3 \end{bmatrix} = \begin{bmatrix} J_1\dot{\omega}_1 + J_\omega \dot{\nu}_1 \\ J_2\dot{\omega}_2 + J_\omega \dot{\nu}_2 \\ J_3\dot{\omega}_3 + J_\omega \dot{\nu}_3 \end{bmatrix} + \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix} \begin{bmatrix} J_1\omega_1 + J_\omega \nu_1 \\ J_2\omega_2 + J_\omega \nu_2 \\ J_3\omega_3 + J_\omega \nu_3 \end{bmatrix}$$

$$M_1 = J_1\dot{\omega}_1 + J_\omega\dot{\nu}_1 - (J_2 - J_3)\omega_2\omega_3 + J_\omega(\omega_2\nu_3 - \omega_3\nu_2)$$

$M_2 = \ldots$

$M_3 = \ldots$

PLUG IN $\dot{\theta}-\omega$ RELATIONSHIP FROM ABOVE & NEGLECT "SMALL" PRODUCTS

$$M_1 = J_1\ddot{\theta}_1 + J_\omega\dot{\nu}_1$$
$$\Rightarrow M_2 = J_2\ddot{\theta}_2 + J_\omega\dot{\nu}_2$$
$$M_3 = J_3\ddot{\theta}_3 + J_\omega\dot{\nu}_3$$

$$\Rightarrow \quad M_1 + u_1 = J_1\ddot{\theta}_1$$
$$M_2 + u_2 = J_2\ddot{\theta}_2$$
$$M_3 + u_3 = J_3\ddot{\theta}_3$$