Single Axis Attitude Control Example: 3/27/08

$J\ddot{\theta} = M$

$J \rightarrow$ mass moment of inertia

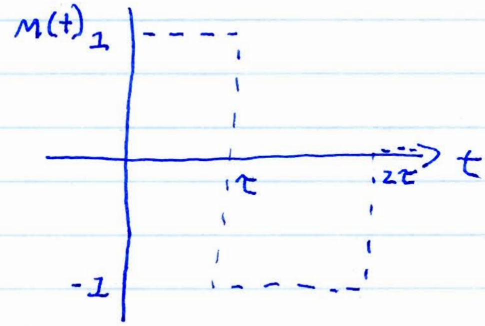
$M \rightarrow$ applied Torque

$\theta(0) = \alpha$

goal: $\theta = 0$

$\dot{\theta}(0) = 0$

$e = \theta_{desired} - \theta(0) = -\alpha$



$$M(t) = c\left(u_s(t) - 2u_s(t-\tau) + u_s(t-2\tau)\right)$$

$$\mathcal{L}\left(f(t-\tau)\right) = e^{-s\tau}F(s)$$

$$M(s) = c\left(\frac{1}{s} - 2e^{-s\tau}\frac{1}{s} + e^{-2s\tau}\frac{1}{s}\right)$$

$$= \frac{c}{s}\left(1 - 2e^{-s\tau} + e^{-2s\tau}\right)$$

$$M(s) = \mathcal{L}(J\ddot{\theta})$$

$$= J\left(s^2\Theta - s\underbrace{\theta(0)}_{\alpha} - \overset{0}{\cancel{\dot{\theta}(0)}}\right)$$

$$\boxed{M(s) = Js^2\Theta - Js\alpha}$$

$$\theta(s) = \frac{\alpha}{s} + \frac{c}{Js^3}\left(1 - 2e^{-s\tau} + e^{-2s\tau}\right)$$

$$\mathcal{L}\left(\frac{1}{s^3}\right) = u_s(t)\left(\frac{t^2}{2}\right)$$

$$\theta(t) = \alpha\, u_s(t) + \frac{c}{J}\left(\frac{t^2}{2}u_s(t) - 2\frac{(t-\tau)^2}{2}u_s(t-\tau) + \frac{(t-2\tau)^2}{2}u_s(t-2\tau)\right)$$

$$\theta(t) = \alpha + \frac{c\tau^2}{J} \quad (t > 2\tau)$$

Problems with Open Loop Control

1) $\theta(0) = \alpha + \Delta\alpha$

$$(\alpha + \Delta\alpha) - \frac{\alpha J (1)^2}{J} = \Delta\alpha$$ small error ∴ not perfectly zero

2) $J = J + \Delta J$ moment of inertia is now off as well

$$\alpha - \frac{\alpha J(1)^2}{J + \Delta J} = \alpha\left(\frac{\Delta J}{J + \Delta J}\right) \neq 0$$

3) Disturbances

$$M(t) = M_c(t) + M_d(t) = \delta(t)$$

$$\theta_d(t) = \mathcal{L}^{-1}\left(\frac{1}{Js^2}\right) = \boxed{\frac{t}{J}}$$ Need to have closed loop to compensate for this

Closed Loop Control 3/27/08

(i) prop. feedback

$$M_c(t) = -K_p\,\theta(t)$$ Marginally stable

$$Js^2\,\Theta(s) - Js\alpha = -K_p\,\Theta(s)$$

$$\boxed{\Theta(s) = \frac{\alpha Js}{Js^2 + K_p} = \frac{\alpha s}{s^2 + \underbrace{K_p/J}_{\omega^2}}}$$ proportional control law

$$\omega = \sqrt{K_p/J}$$

$$\theta(t) = \alpha\cos\omega t$$

(ii) $$\boxed{M_c(t) = -K_p\,\theta(t) - K_d\,\dot\theta(t)}$$

$$Js^2\,\Theta(s) - Js\alpha = -\Theta(s)\,(K_p + sK_d) + K_d\alpha$$

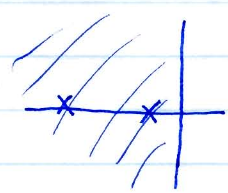
$$\Theta(s) = \frac{\alpha(Js + K_d)}{Js^2 + K_d s + K_p} \rightarrow s^2 + 2\zeta\omega_n s + \omega_n^2$$

$$\boxed{s = \frac{-K_d \pm \sqrt{K_d^2 - 4JK_p}}{2J}}$$

$$\boxed{= -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1}}$$

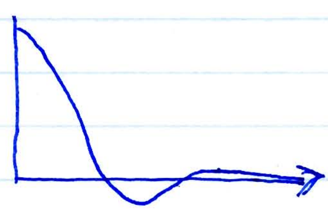
overdamped: $\zeta > 1$

$K_d^2 > 4JK_p$



stable

$$\theta(t) = K_1 e^{s_1 t} + K_2 e^{s_2 t}$$

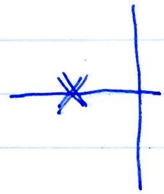


Real distinct Roots

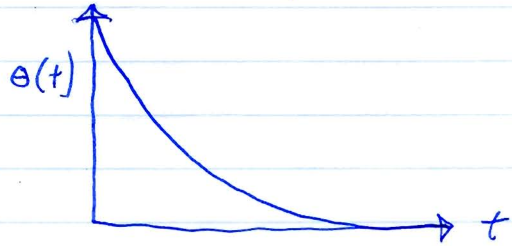
critically damped: 3/27/08

$\zeta = 1$ $\qquad s_1 = s_2$

$k_d^2 = 4Jk_p$



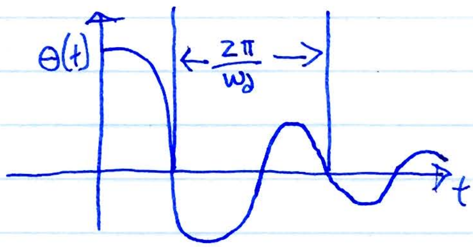
$\theta(t) = k_1 e^{st} + k_2 t e^{st}$



Underdamped:

$\zeta < 1 \qquad k_d^2 < 4Jk_p$

$\theta(t) = k_1 e^{-\frac{k_d}{2}t} \sin(\omega_d t + \theta)$

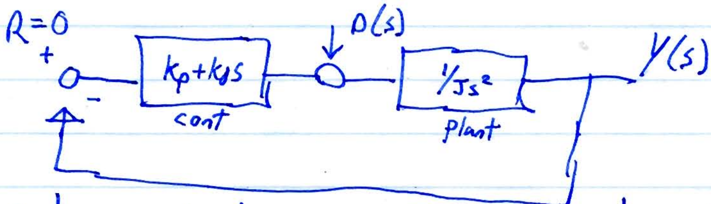


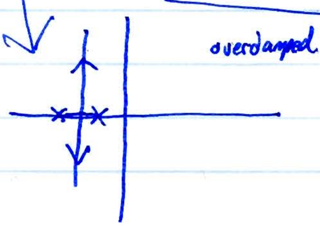
Assume

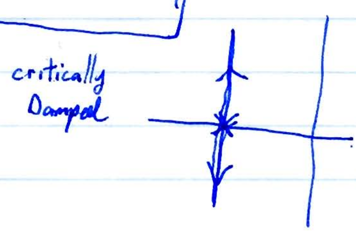
$k_d = 1 \quad J = 1$

Root locus

$$\frac{\theta(s)}{D(s)} = \frac{1}{s^2 + k_d s + k_p}$$







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Assume $D=0$

$$\boxed{\frac{Y(s)}{R(s)} = \frac{k_p + k_d s}{s^2 + k_d s + k_p}}$$