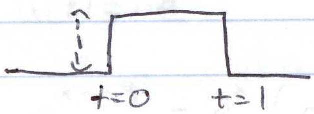
LaPlace Transform Basics

$\dot{y} + ky = u$

output $\quad$ ↑input

$u(t) = a_1 u_1(t) + a_2 u_2(t)$



$u(t) = 1\, u_s(t) - 1\, u_s(t-1)$

$y(t) = a_1 y_1(t) + a_2 y_2(t)$

$\dot{y}_1 + k y_1 = u_1$

$\dot{y}_2 + k y_2 = u_2$

$(a_1 \dot{y}_1 + a_2 \dot{y}_2) + k(a_1 y_1 + a_2 y_2) = a_1(\dot{y}_1 + k y_1) + a_2(\dot{y}_2 + k y_2)$

$= a_1 u_1 + a_2 u_2 = u$

$$\int_{t=a}^{t=b} \delta(t) f(t)\, dt = f(0)$$

$a<0, \; b>0$

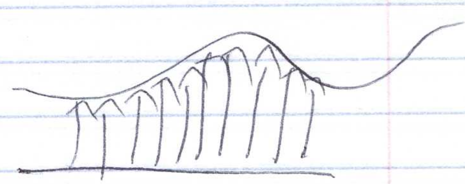
$$\int_{0^-}^{b>0} \delta(t) f(t)\, dt = f(0)$$

$$\int_{0^+}^{b>0} \delta(t) f(t)\, dt = 0$$

$\cancel{u(t) = \int_{-\infty}^{\infty} \delta(t-\tau)\, u(t-\tau)\, d\tau}$

"a"

$$u(t) = \int_{-\infty}^{\infty} \delta(t-\tau)\, u(\tau)\, d\tau$$



$h(t)$ is the response to $\delta(t)$

$h(t-\tau)$ is the response to $\delta(t-\tau)$

$$y(t) = \int_{-\infty}^{\infty} h(t-\tau)\, u(\tau)\, d\tau \Rightarrow \text{"convolution"}$$

$$y = h * u = u * h$$

$$u(t) = e^{st} \qquad s = x + jy$$

$$e^{x+jy} = e^{x}(\cos y + j \sin y)$$

$$a\cos(\omega t) = \frac{a}{2}\big((\cos\omega t + j\sin\omega t) + (\cos\omega t - j\sin\omega t)\big)$$

$$= \frac{a}{2}\left(e^{-j\omega t} + e^{j\omega t}\right)$$

$$u(t) = e^{st}$$

$$y(t) = \int_{-\infty}^{\infty} h(\tau)\, e^{s(t-\tau)}\, d\tau$$

$$= e^{st} \underbrace{\int_{-\infty}^{\infty} h(\tau) e^{-s\tau}\, d\tau}_{\substack{\text{independent of time} \\ H(s)}}$$

$$y(t) = a\,|H(j\omega)|\cos(\omega t + \angle H(j\omega))$$