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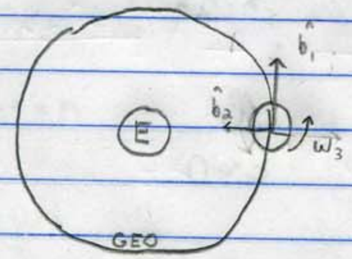
Today's satellite: Intelsat IV (possibly A)

spin stabilized, but not all sections spin

"dual spinner"



$$\omega_3 = \frac{360°}{24 \cdot 3600} = .004167 \ \frac{deg}{s}$$

$$\approx 7.3 \cdot 10^{-5} \ rad/s$$

b frame rotates w/ platform & not w/ rotor

passively stabilized about the minor axis

↳ nutation damping on de-spun section

$${}^{A}\vec{\omega}^{B} = \omega_1 \hat{b}_1 + \omega_2 \hat{b}_2 + \omega_3 \hat{b}_3 \qquad {}^{A}\vec{\omega}^{C} = \omega_1 \hat{b}_1 + \omega_2 \hat{b}_2 + \Omega \hat{b}_3$$

J_a - axial component of platform

J_r - " " " rotor

J_t - transverse for both

$$\cancel{M}^{0} = \dot{H} + S(\omega) H$$

$$H = \begin{bmatrix} J_t \omega_1 \\ J_t \omega_2 \\ J_a \omega_3 + J_r \Omega \end{bmatrix} \qquad \dot{H} = \begin{bmatrix} J_t \dot{\omega}_1 \\ J_t \dot{\omega}_2 \\ J_a \dot{\omega}_3 + J_r \dot{\Omega} \end{bmatrix}$$

$$0 = J_t \dot{\omega}_1 - (J_t - J_a)\omega_2 \omega_3 + J_r \omega_2 \Omega$$
$$0 = J_t \dot{\omega}_2 + (J_t - J_a)\omega_1 \omega_3 - J_r \omega_1 \Omega$$
$$0 = J_a \dot{\omega}_3 + J_r \dot{\Omega}$$

going to assume Ω = fixed, $\Rightarrow \dot{\Omega} = 0$

$\Rightarrow \omega_3 = \text{constant} = n$

$$\begin{cases} 0 = \dot{\omega}_1 - \omega_2 \left[\dfrac{(J_t - J_a) n + J_r \Omega}{J_t} \right] \\[2ex] 0 = \dot{\omega}_2 + \omega_1 \left[\dfrac{(J_t - J_a) n - J_r \Omega}{J_t} \right] \end{cases}$$

→ "λ_p"

becomes $\begin{cases} 0 = \dot{\omega}_1 - \lambda_p \omega_2 \\ 0 = \dot{\omega}_2 + \lambda_p \omega_1 \end{cases}$

$$\omega_1(t) = \omega_1(0)\cos(\lambda_p t) + \omega_2(0)\sin(\lambda_p t)$$
$$\omega_2(t) = \omega_2(0)\cos(\lambda_p t) - \omega_1(0)\sin(\lambda_p t)$$

$T = \frac{1}{2}\omega^T H = \frac{1}{2}\omega^T J \omega$ (doesn't really work any more)

mass in rotor not rotating w/ same ω

$$T = \frac{1}{2}(J_t \omega_t^2 + J_a \omega_3^2 + J_r \Omega^2)$$
$$\dot{T} = J_t \omega_t \dot{\omega}_t + J_a \omega_3 \dot{\omega}_3 + J_r \Omega \dot{\Omega} \quad < 0$$

which would seem to contradict our initial analysis

$$H^2 = J_t^2 \omega_t^2 + (J_a \omega_3 + J_r \Omega)^2$$
$$2H\dot{H} = 0 = 2J_t^2 \omega_t \dot{\omega}_t + 2(J_a\omega_3 + J_r\Omega)(J_a\dot{\omega}_3 + J_r\dot{\Omega})$$

$$J_t \omega_t \dot{\omega}_t = -\frac{1}{J_t}(J_a\omega_3 + J_r\Omega)(J_a\dot{\omega}_3 + J_r\dot{\Omega})$$

$$\dot{T} = J_a\dot{\omega}_3\omega_3 + J_r\dot{\Omega}\Omega - \frac{1}{J_t}(J_a\omega_3 + J_r\Omega)(J_a\dot{\omega}_3 + J_r\dot{\Omega})$$
$$= J_a\dot{\omega}_3\Big(\underbrace{\omega_3 - \frac{J_a\omega_3 + J_r\Omega}{J_t}}_{\text{"}\lambda_p\text{"}}\Big) + J_r\dot{\Omega}\Big(\underbrace{\Omega - \frac{J_a\omega_3 + J_r\Omega}{J_t}}_{\text{"}\lambda_r\text{"}}\Big)$$

$$\dot{T} = \underbrace{\lambda_p J_a \dot{\omega}_3}_{\text{"}\dot{T}_p\text{"}} + \underbrace{\lambda_r J_r \dot{\Omega}}_{\text{"}\dot{T}_r\text{"}}$$

$$\Rightarrow J_a\dot{\omega}_3 = \dot{T}_p/\lambda_p \qquad J_r\dot{\Omega} = \dot{T}_r/\lambda_r$$

$$\rightarrow J_t\omega_t\dot{\omega}_t = \underbrace{-\frac{1}{J_t}(J_a\omega_3 + J_r\Omega)}_{\text{"}\lambda_0\text{"}}(\dot{T}_p/\lambda_p + \dot{T}_r/\lambda_r)$$

$\sin\theta = \frac{H_t}{H} = \frac{J_t\omega_t}{H}$

$\dot{\theta}\cos\theta = J_t\dot{\omega}_t / H \quad \Rightarrow \quad \dot{\theta}\cos\theta = \frac{J_t\dot{\omega}_t\omega_t}{H^2\sin\theta / J_t}$

$\Rightarrow \dot{\theta} = \frac{-J_t\lambda_0}{H^2\sin\theta\cos\theta}\left(\frac{\dot{T}_p}{\lambda_p} + \frac{\dot{T}_r}{\lambda_r}\right)$

For stability, $\dot{\theta} < 0$ i.e. $\frac{\dot{T}_p}{\lambda_p} + \frac{\dot{T}_r}{\lambda_r} > 0$

Assume $\omega_3 << \Omega$ i.e. $\omega_3 \approx 0$

$\lambda_p \approx -\frac{J_r\Omega}{J_t} < 0 \qquad \lambda_r \approx \Omega\left(1 - \frac{J_r}{J_t}\right)$

If $J_r > J_t$, $\lambda_r < 0$

" $J_r < J_t$ $\lambda_r > 0$

If λ_r & λ_p are < 0, both $\dot{T}_p$ & $\dot{T}_r$ may be negative

↳ always stable

If $J_r < J_t$ (Intelsat) energy dissipation in the rotor is destabilizing

to stabilize, must make $\left|\frac{\dot{T}_p}{J_r/J_t}\right| > \left|\frac{\dot{T}_r}{1 - \frac{J_r}{J_t}}\right|$