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$$T = \frac{1}{2}\left(J_{11}\omega_1^2 + J_{22}\omega_2^2 + J_{33}\omega_3^2\right)$$

$$= \frac{1}{2}\left(J_t\omega_t^2 + J_a\omega_3^2\right)$$

spins are stable about major and minor axis and unstable about the intermediate axis

Eqs. of motion

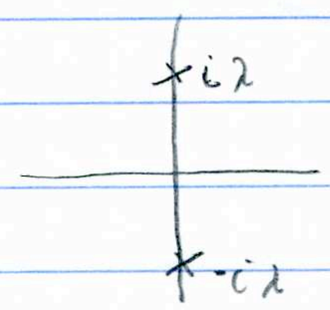
$\dot{\omega}_1 - \lambda\omega_2 = 0$

$\dot{\omega}_2 + \lambda\omega_1 = 0$

roots $\rightarrow s^2 + \lambda^2$

$(i\lambda)^2 + \lambda^2 = 0$

$\uparrow \pm i\lambda \rightarrow$ poles



$\rightarrow$ neutrally stable for minor axis

$$\cos^2\theta = \left(\frac{H_a}{H}\right)^2 = \frac{J_a^2\omega_3^2}{H^2}$$

$$\sin^2\theta = \left(\frac{H_t}{H}\right)^2 = \frac{J_t^2\omega_t^2}{H^2}$$

$$T = \frac{1}{2}\left(\frac{J_t^2\omega_t^2}{J_t} + \frac{J_a^2\omega_3^2}{J_a}\right)\frac{H^2}{H^2}$$

$$T = \frac{H^2}{2}\left(\frac{J_t^2\omega_t^2}{H^2 J_t} + \frac{J_a^2\omega_3^2}{H^2 J_a}\right)$$

$$= \frac{H^2}{2}\left(\frac{\sin^2\theta}{J_t} + \frac{\cos^2\theta}{J_a}\right)$$

$$= \frac{H^2}{2J_a}\left(\frac{J_a}{J_t}\sin^2\theta + 1 - \sin^2\theta\right)$$

$$= \frac{H^2}{2J_a}\left(1 + \sin^2\theta\left(\frac{J_a}{J_t} - 1\right)\right)$$

$$T = \frac{H^2}{2J_a}\left(1 + \sin^2\theta\left(\frac{J_a - J_t}{J_t}\right)\right)$$

$$\dot{T} = \frac{H^2}{2J_a}\left(2\sin\theta\cos\theta\,\dot{\theta}\left(\frac{J_a - J_t}{J_t}\right)\right)$$

What if $\dot{T} < 0$?

$$\dot{\theta}(J_a - J_t) < 0$$

Prolate $\rightarrow$ $\dot{\theta} > 0$ $\Rightarrow$ soup can
oblate $\rightarrow$ $\dot{\theta} < 0$ $\rightarrow$ tuna can

$$\vec{f} = -k\dot{x}\hat{x}$$

$$dW = -k\dot{x}\,dx = -k\dot{x}^2\,dx$$

$$W_{av} = \left(\int_0^p -k\dot{x}^2\,dx\right)\frac{1}{p}$$

$$= -\frac{k}{p}\int_0^p \dot{x}^2\,dx$$

$$W_{av} = -c\,\theta^2$$

$$\dot{T} = W_{av}$$

$$\frac{H^2}{J_a}\left(\frac{J_a - J_t}{J_t}\right)\sin\theta\cos\theta\,\dot{\theta} = -c\,\theta^2$$

$$\frac{H^2}{J_a}\left(\frac{J_a - J_t}{J_t}\right)\cancel{\theta}\,\dot{\theta} = -c\,\theta^{\cancel{2}}$$

$$\dot{\theta} + \frac{1}{\tau}\theta = 0$$

$$\tau = \frac{H^2}{J_a}\left(\frac{J_a - J_t}{J_t}\right)\frac{1}{c}$$

$$\theta(t) = \theta_0 e^{-t/\tau}$$

$$H^2 = J_t^2 w_t^2 + J_a^2 w_3^2$$

$$H(\text{final}) = J_a w_3(\text{final})$$

$$J_a w_3(\text{final}) = \sqrt{J_t^2 w_t^2 + J_a^2 w_3^2}$$

$$w_3(\text{final}) = \sqrt{\left(\frac{J_t}{J_a}\right)^2 w_t^2 + w_3^2}$$

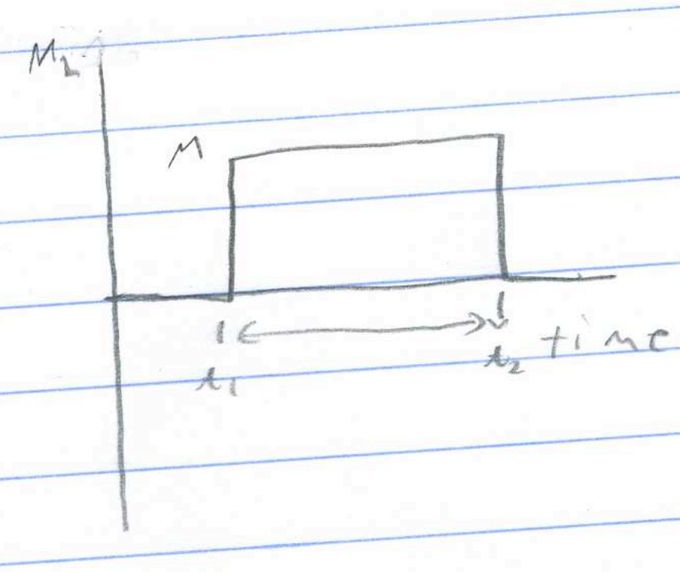
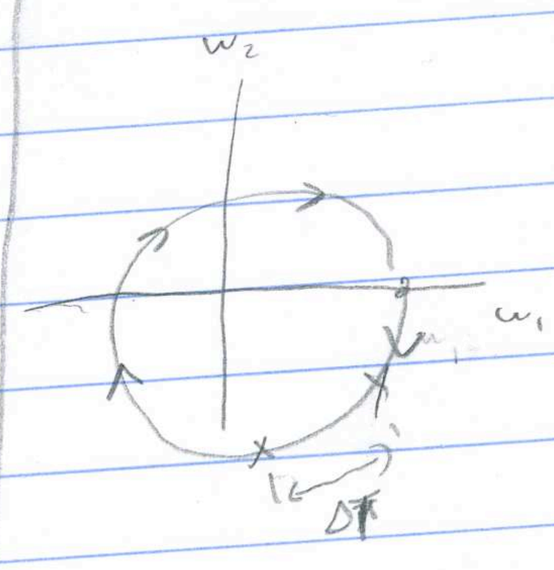
$$M_1 = J_t \dot{w}_1 - (J_t - J_a) w_3 w_2$$

$$M_2 = J_t \dot{w}_2 + (J_t - J_a) w_3 w_1$$

$$\boxed{0 = J_a \dot{w}_3} \rightarrow \quad w_3(t) = n \Rightarrow \text{constant}$$

$$\frac{M_1}{J_t} = \dot{w}_1 - \left(\frac{J_t - J_a}{J_t}\right) n w_2$$

$$\frac{M_2}{J_t} = \dot{w}_2 + \left(\frac{J_t - J_a}{J_t}\right) n w_1$$

$$\frac{M_1}{J_t} = \dot{w}_1 - \lambda w_2$$

$$\frac{M_2}{J_t} = \dot{w}_2 + \lambda w_1$$