from last time

$$\dot{\omega} = J^{-1}(M - S(\omega) J\omega)$$

skew semetric matrix

inertia matrix

$$\dot{Q} = \tfrac{1}{2}\Omega \otimes Q$$

Kinematic differential eqs. of motion

$$\vec{L} = m\,{}^A\vec{V}_c$$

K.E. $\rightarrow T_{translational} = \frac{1}{2} m\,{}^A\vec{V} \cdot {}^A\vec{V} = \frac{1}{2} m\,|\vec{V}_c|^2$

$$= \frac{1}{2}\,{}^A\vec{V}_c \cdot (m\,{}^A\vec{V}_c) = \frac{1}{2}\,{}^A\vec{V}_c \cdot \vec{L}$$

$$T_{rotational} = \underbrace{\frac{1}{2}\,{}^A\vec{\omega}^B \cdot \vec{H}}_{\frac{1}{2}\omega^T J \omega}$$

$$J\dot{\omega} = M - S(\omega) J \omega$$

$$J = \begin{bmatrix} J_1 & 0 & 0 \\ 0 & J_2 & 0 \\ 0 & 0 & J_3 \end{bmatrix}$$

$$M_1 = J_1 \dot{\omega}_1 - (J_2 - J_3)\,\omega_2 \omega_3$$

$$M_2 = J_2 \dot{\omega}_2 - (J_3 - J_1)\,\omega_3 \omega_1$$

$$M_3 = J_3 \dot{\omega}_3 - (J_1 - J_2)\,\omega_1 \omega_2$$

Torque Free

$M_1 = M_2 = M_3 = 0$

Axi symatric matrix

$J_1 = J_2 = J_t$

$J_3 = J_a$

$0 = J_t \dot{w}_1 - (J_t - J_a) w_2 w_3$

$0 = J_t \dot{w}_2 - (J_a - J_t) w_3 \cdot w_1$

$0 = J_a \dot{w}_3 - \cancel{(J_t - J_t)}^{\to 0} w_1 w_2$

$w_3 =$ constant $= n =$ "spin rate"

$0 = J_t \dot{w}_1 - (J_t - J_a) n w_2$

$= J_t \dot{w}_2 + (J_t - J_a) n w_1$

relative spin rate

$$\lambda = \frac{(J_t - J_a) n}{J_t}$$

$0 = \dot{w}_1 - \lambda w_2$

$0 = \dot{w}_2 - \lambda w_1$

$0 = w_1 \dot{w}_1 - \lambda w_1 w_2$

$+ \quad 0 = w_2 \dot{w}_2 + \lambda w_1 w_2$

$0 = w_1 \dot{w}_1 + w_2 \dot{w}_2$

$$\omega_1^2 + \omega_2^2 = \text{constant}$$

$$2\omega_1\dot{\omega}_1 + 2\omega_2\dot{\omega}_2 = 0$$

$$\omega_t^2 = \omega_1^2 + \omega_2^2$$

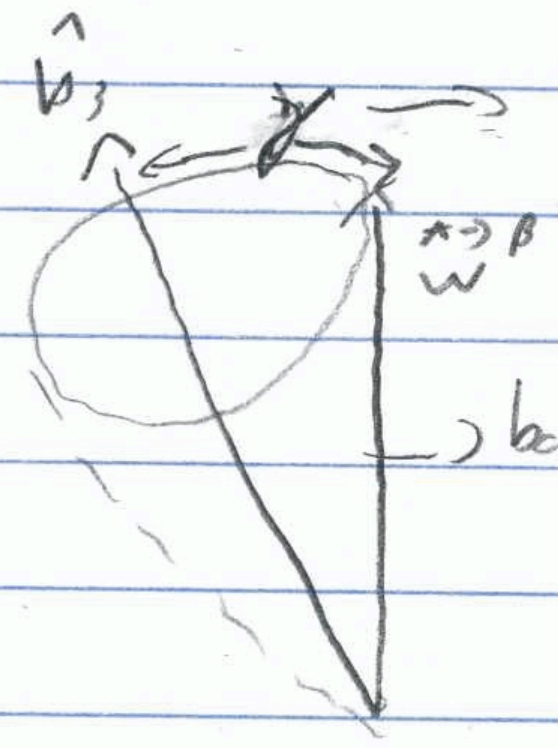
$$|{}^{A\to B}\omega|^2 = \omega_t^2 + n^2$$

$$T = \frac{1}{2}\omega^T J \omega$$

$$= \frac{1}{2}\begin{bmatrix}\omega_1 & \omega_2 & \omega_3\end{bmatrix}\begin{bmatrix}J_t & 0 & 0\\ 0 & J_t & 0\\ 0 & 0 & J_a\end{bmatrix}\begin{bmatrix}\omega_1\\ \omega_2\\ \omega_3\end{bmatrix}$$

$$= \frac{1}{2}\left(J_t\omega_1^2 + J_t\omega_2^2 + J_a\omega_3^2\right)$$

$$= \frac{1}{2}\left(J_t\omega_t^2 + J_a n^2\right)$$



$$\tan\gamma = \frac{\omega_t}{n}$$

$$H = \begin{bmatrix}J_t\omega_1\\ J_t\omega_2\\ J_a n\end{bmatrix}$$

$$H_t = \sqrt{H_1^2 + H_2^2}$$

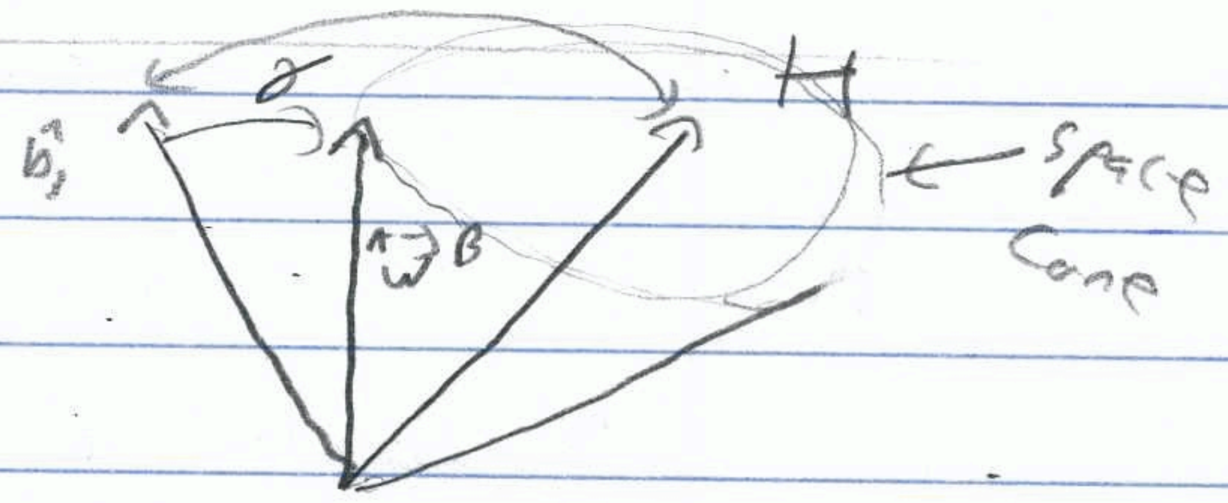
$$= \sqrt{J_t^2(\omega_1^2 + \omega_2^2)}$$

$$= J_t\omega_t$$

$$H_a = J_a n$$

$$\tan\gamma = \frac{\omega_t}{n}$$

wobble



$$\tan\theta = \frac{H_t}{H_a} = \frac{J_t\omega_t}{J_a n}$$

direct procession

$\theta > \gamma$ or $\tan\theta > \tan\gamma$

$$\frac{J_t\omega_t}{J_a n} > \frac{\omega_t}{n}$$

$J_t > J_a \Rightarrow$ "prolate"

soup can