2/19 Another gravity gradient satellite like Transit, but not in 60's in 80.

Polar BEAR: Beacon Experiment + Auroral Research

rather than permanent magnets → momentum wheel for pitch axis

3 months after launch Solar Radiation flipped the satellite over and it re-stabilized upside down

to fix they stopped momentum wheel, destabilized, respun the wheel and it flipped back over to the correct position

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$H = J\omega$

3x1 B frame; J: B frame; ω: B w/respect to inertial Frame

ex: $\begin{bmatrix}1\\2\\3\end{bmatrix} = \begin{bmatrix}1&0&0\\0&2&0\\0&0&3\end{bmatrix}\begin{bmatrix}1\\1\\1\end{bmatrix}$

H + ω do not point in the same direction unless $\lambda\omega = J\omega$

principal moments of Inertia (Eigen values): $\det(\lambda I - J) = 0$

Principal axes (Eigen vectors): $(\lambda_i I - J)e_i = 0$

see example

Now we have the B frame and E frame (principal axes)

$$R^{E/B} = \begin{bmatrix}e_{11}&e_{12}&e_{13}\\e_{21}&e_{22}&e_{23}\\e_{31}&e_{32}&e_{33}\end{bmatrix} = \begin{bmatrix}-1/\sqrt{2}&1/\sqrt{2}&0\\0&0&1\\1/\sqrt{2}&1/\sqrt{2}&0\end{bmatrix}$$

$H_E = R^{E/B} H_B \qquad \omega_E = R^{E/B}\omega_B \rightarrow \omega_B = (R^{E/B})^T \omega_E$

$$\begin{aligned}H_E &= R^{E/B} H_B\\ &= R^{E/B} J_B \omega_B\\ &= \boxed{R^{E/B} J_B (R^{E/B})^T}\,\omega_E\end{aligned}$$

$$J_E = R^{E/B} J_B (R^{E/B})^T \quad \text{diagonalization}$$

$H_E = J_E \omega_E$

$$\begin{bmatrix}-1/\sqrt{2}&1/\sqrt{2}&0\\0&0&1\\1/\sqrt{2}&1/\sqrt{2}&0\end{bmatrix}\begin{bmatrix}2&1&0\\1&2&0\\0&0&2\end{bmatrix}\begin{bmatrix}-1/\sqrt{2}&0&1/\sqrt{2}\\1/\sqrt{2}&0&1/\sqrt{2}\\0&1&0\end{bmatrix}$$

$$= \begin{bmatrix}-1/\sqrt{2}&1/\sqrt{2}&0\\0&0&2\\3/\sqrt{2}&3/\sqrt{2}&0\end{bmatrix}\begin{bmatrix}-1/\sqrt{2}&0&1/\sqrt{2}\\1/\sqrt{2}&0&1/\sqrt{2}\\0&1&0\end{bmatrix}$$

$$= \begin{bmatrix}1&0&0\\0&2&0\\0&0&3\end{bmatrix}$$

Transform 1 moment of inertia matrix to another using Rotation Matrix

$\vec{F} = \frac{{}^{A}d\vec{L}}{dt}$

$\vec{M} = \frac{{}^{A}d\vec{H}}{dt} = \frac{{}^{B}d\vec{H}}{dt} + {}^{A}\vec{\omega}^{B} \times \vec{H}$

$$\overbrace{M = \dot{H} + S(\omega)H}^{\text{All in frame B}}$$

$$\begin{bmatrix} M_1 \\ M_2 \\ M_3 \end{bmatrix} = \begin{bmatrix} \dot{H}_1 \\ \dot{H}_2 \\ \dot{H}_3 \end{bmatrix} + \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix} \begin{bmatrix} H_1 \\ H_2 \\ H_3 \end{bmatrix}$$

$M = \dot{H} + S(\omega) J\omega$ $\quad \dot{H} = J\dot{\omega} + \cancel{\dot{J}}\omega$

rigid body

$M = J\dot{\omega} + S(\omega)J\omega$ $\quad$ J is positive definite → Invertible

$$\boxed{\dot{\omega} = J^{-1}(M - S(\omega)J\omega)} \longleftrightarrow \boxed{\dot{Q} = \frac{1}{2}\Omega \otimes Q}$$

Finding principal axes

For a moment of inertia matrix J written with respect to frame B, each root $\lambda_1, \lambda_2, \lambda_3$ of

$$\det(\lambda I - J) = 0$$

is a principal moment of inertia, and each vector e_i satisfying

$$(\lambda_i I - J)e_i = 0$$

defines the coordinates of a principal axis with respect to frame B. For example, consider the inertia matrix

$$J = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}.$$

1. Find the principal moments of inertia. (Work this out by hand.)

$\cancel{2(4-0) - 1(2-0) + 0}$

$$\begin{bmatrix} \lambda-2 & -1 & 0 \\ -1 & \lambda-2 & 0 \\ 0 & 0 & \lambda-2 \end{bmatrix}$$

$\cancel{(\lambda-2)((\lambda-2)(\lambda-2)-0) - 1(\lambda-2-0)+0}$

$\cancel{(\lambda^2-4\lambda+4)(\lambda-2) - \lambda+2}$

$\cancel{\lambda^3-6\lambda^2+12\lambda-8-\lambda+2}$

$\cancel{\lambda^3-6\lambda^2+11\lambda-6=0}$

$0 - 0 + \lambda-2((\lambda-2)(\lambda-2)-1)$

$(\lambda-2)(\lambda^2-4\lambda+3)=0$

$(\lambda-3)(\lambda-1)$

$\cancel{\lambda_1=2}$ $\cancel{\lambda_2=3}$ $\cancel{\lambda_3=1}$

$\lambda_1=1$ $\lambda_2=2$ $\lambda_3=3$

2. Find the principal axis corresponding to the minor moment of inertia. (Again, do this by hand.)

$\cancel{\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} - \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}} \cancel{\begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} = 0}$

$\cancel{y_1 - x_1 = 0 \quad x_1 = y_1 \quad e_1 = (\tfrac{1}{\sqrt{2}}, \tfrac{1}{\sqrt{2}}, 0)}$

or make e_1 $\begin{bmatrix} -1/\sqrt{2} \\ 1/\sqrt{2} \\ 0 \end{bmatrix}$

so make e_3 $\begin{bmatrix} -1/\sqrt{2} \\ -1/\sqrt{2} \\ 0 \end{bmatrix}$

$e_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ $Je_1 = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \\ 0 \end{bmatrix}$ $\lambda_1 e_1 = 1\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \neq \begin{bmatrix} 3 \\ 3 \\ 0 \end{bmatrix}$

$(\lambda_1 I - J)e_1 = 0$

$$\left(\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}\right)\begin{bmatrix} e_{11} \\ e_{12} \\ e_{13} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -1 & -1 & 0 \\ -1 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}\begin{bmatrix} e_{11} \\ e_{12} \\ e_{13} \end{bmatrix}$$

$-e_{11} = e_{12}$

$e_{13} = 0$ $\rightarrow \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$

$\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$ ✓ normalize: $\begin{bmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \\ 0 \end{bmatrix}$

$e_2: \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$e_3: \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \\ 0 \end{bmatrix}$

$\omega \times r = S(\omega) r$

$S(e_1)e_2 =$

$$\begin{bmatrix} 0 & -e_{13} & e_{12} \\ e_{13} & 0 & -e_{11} \\ -e_{12} & e_{11} & 0 \end{bmatrix}\begin{bmatrix} e_{21} \\ e_{22} \\ e_{23} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & -1/\sqrt{2} \\ 0 & 0 & \frac{1}{\sqrt{2}} \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -1/\sqrt{2} \\ 1/\sqrt{2} \\ 0 \end{bmatrix} = -e_3$$