Satellite: Similar to TRANSIT, gravity-gradient stability, launched in late 80s

Polar BEAR: Beacon Experiment and Auroral Research

3x1 * coordinates always in frame B

$$H = J\omega = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \qquad \underbrace{\lambda}_{\substack{\text{Eigenvalue of } J \\ = \text{principal MOI}}} \omega = J \underbrace{\omega}_{\substack{\text{Eigenvector of } J \\ = \text{principal axes}}}$$

Principal MOI: $\det(\lambda I - J) = 0$, Principal axes: $(\lambda_i I - J)e_i = 0$

Handout Example:

$$J = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$ Find principal MOI + principal axis for minor MOI

$\lambda = 1, 2, 3$

$$(\lambda_1 I - J)e_1 = 0 \Rightarrow \left(\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} \right) \begin{bmatrix} e_{11} \\ e_{12} \\ e_{13} \end{bmatrix} = 0$$

$$\begin{bmatrix} -1 & -1 & 0 \\ -1 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} e_{11} \\ e_{12} \\ e_{13} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$-e_{11} - e_{12} = 0$, $-e_{11} - e_{12} = 0$, $-e_{13} = 0$

$\Rightarrow e_{11} = -e_{12}$, $e_{13} = 0$ $\Rightarrow \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$

$$\lambda_1 e_1 = 1 \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} \qquad J e_1 = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} \checkmark$$

$$e_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad e_3 = \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \\ 0 \end{bmatrix}$$ for convention $e_1 = \begin{bmatrix} 1/\sqrt{2} \\ -1/\sqrt{2} \\ 0 \end{bmatrix}$

$\vec{\omega} \times \vec{r} \longleftrightarrow S(\omega) r$

$$S(e_1)e_2 = \begin{bmatrix} 0 & -e_{13} & e_{12} \\ e_{13} & 0 & -e_{11} \\ -e_{12} & e_{11} & 0 \end{bmatrix} \begin{bmatrix} e_{21} \\ e_{22} \\ e_{23} \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1/\sqrt{2} \\ 0 & 0 & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -1/\sqrt{2} \\ -1/\sqrt{2} \\ 0 \end{bmatrix} = -e_3$$

* want to end up with a coord. system that's right handed: take $-e_1$

Frame E: lined up with principal axes

$$R^{E/B} = \begin{bmatrix} e_{11} & e_{12} & e_{13} \\ e_{21} & e_{22} & e_{23} \\ e_{31} & e_{32} & e_{33} \end{bmatrix} = \begin{bmatrix} -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \end{bmatrix}$$

$$H_B = J_B \omega_B \Rightarrow H_E = R^{E/B} H_B, \quad \omega_E = R^{E/B} \omega_B$$

$$\omega_B = (R^{E/B})^T \omega_E$$

$$H_E = R^{E/B} H_B = R^{E/B} J_B \omega_B = R^{E/B} J_B (R^{E/B})^T \omega_E = J_E \omega_E$$

$$\Rightarrow J_E = R^{E/B} J_B (R^{E/B})^T$$

$$\begin{bmatrix} -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \end{bmatrix} \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} -1/\sqrt{2} & 0 & 1/\sqrt{2} \\ 1/\sqrt{2} & 0 & 1/\sqrt{2} \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 2 \\ 3/\sqrt{2} & 3/\sqrt{2} & 0 \end{bmatrix} \begin{bmatrix} -1/\sqrt{2} & 0 & 1/\sqrt{2} \\ 1/\sqrt{2} & 0 & 1/\sqrt{2} \\ 0 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$$\vec{F} = {}^A\frac{d\vec{L}}{dt} \qquad \vec{M} = {}^A\frac{d\vec{H}}{dt} = {}^B\frac{d\vec{H}}{dt} + {}^A\vec{\omega}^B \times \vec{H}$$

$$M = \dot{H} + S(\omega) H \Rightarrow \begin{bmatrix} M_1 \\ M_2 \\ M_3 \end{bmatrix} = \begin{bmatrix} \dot{H}_1 \\ \dot{H}_2 \\ \dot{H}_3 \end{bmatrix} + \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix} \begin{bmatrix} H_1 \\ H_2 \\ H_3 \end{bmatrix}$$

* all in frame B

$$M = \dot{H} + S(\omega) J\omega \qquad \text{where } \dot{H} = J\dot{\omega} + \cancel{\dot{J}\omega}^{\,0} \text{ rigid-body}$$

$$M = J\dot{\omega} + S(\omega) J\omega$$

$$\boxed{\dot{\omega} = J^{-1}(M - S(\omega) J\omega)} \qquad \text{Euler's Eqns.}$$