

$$\dot{Q} = \frac{1}{2} \Omega \otimes Q = \frac{1}{2} \begin{bmatrix} 0 & \omega_3 & -\omega_2 & \omega_1 \\ -\omega_3 & 0 & \omega_1 & \omega_2 \\ \omega_2 & -\omega_1 & 0 & \omega_3 \\ -\omega_1 & -\omega_2 & -\omega_3 & 0 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix}$$

$$\omega_1 = \omega_3 = 2, \omega_2 = 0$$

$$\dot{Q} = \underbrace{\begin{bmatrix} 0 & 1 & 0 & 1 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \\ -1 & 0 & -1 & 0 \end{bmatrix}}_A Q$$

$$Q(t) = e^{At} Q(0)$$

$$\begin{aligned} \dot{q}_1 &= q_2 + q_4 \rightarrow sQ_1 - q_1(0) = Q_2 + Q_4 \\ \dot{q}_2 &= -q_1 + q_3 \rightarrow sQ_2 - q_2(0) = -Q_1 + Q_3 \\ \dot{q}_3 &= -q_2 + q_4 \rightarrow sQ_3 - q_3(0) = -Q_2 + Q_4 \\ \dot{q}_4 &= -q_1 - q_3 \rightarrow sQ_4 - q_4(0) = -Q_1 - Q_3 \end{aligned}$$

$$Q = ( ) q_i(0)$$

$$QT = \int^{-1} ((sI - A)^{-1}) Q(0)$$

## Rigid Body Dynamics (rotational)

$$\vec{F} = \frac{d^A \vec{L}}{dt}$$

$$\vec{M} = \frac{d^A \vec{H}}{dt}$$

$$\begin{aligned} \vec{L} &= \int \vec{r} \times \vec{v} dm \quad \frac{d^A}{dt} (\vec{r}_c + \vec{r}) \\ &= m^A \vec{v}_c \end{aligned}$$

$$\vec{H}_c = \int \vec{r} \times \vec{v} dm$$

$${}^A \vec{v} = \vec{v} + {}^{AB} \omega \times \vec{r}$$

$$\vec{H}_c = \int \vec{r} \times \vec{v} dm + \int \vec{r} \times ({}^{AB} \omega \times \vec{r}) dm$$

$${}^{AB} \omega = \omega_1 \hat{b}_1 + \omega_2 \hat{b}_2 + \omega_3 \hat{b}_3$$

$$\vec{r} = r_1 \hat{b}_1 + r_2 \hat{b}_2 + r_3 \hat{b}_3$$

$$\begin{bmatrix} H_1 \\ H_2 \\ H_3 \end{bmatrix} = \underbrace{\begin{bmatrix} J_{11} & J_{12} & J_{13} \\ J_{21} & J_{22} & J_{23} \\ J_{31} & J_{32} & J_{33} \end{bmatrix}}_J \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix}$$

$$J_{11} = \int (r_2^2 + r_3^2) dm \leftarrow \text{moment of inertia etc.}$$

$$J_{12} = J_{21} = - \int r_1 r_2 dm \leftarrow \text{product of inertia etc.} \quad \text{major}$$

$$J = \begin{bmatrix} 1 & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & 3 \end{bmatrix} \leftarrow \text{if } J_{12} = J_{21} = \dots = 0 \text{ then } J_{11}, J_{22}, J_{33}$$

minor  $\uparrow$  major

$\hat{b}_1$   
intermediate

$$w = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad H = Jw = \begin{bmatrix} 1 \\ a \\ 3 \end{bmatrix}$$

$$\vec{w} \in B = \hat{b}_1, \hat{b}_2, \hat{b}_3$$

$$\vec{H} = \hat{b}_1 + a\hat{b}_2$$