AE 403 - 2/7/08

SAR Satellite - maps surface of Venus
- takes pictures of same area from slightly different angles
$\Rightarrow$ 3-d image

Magellan — Synthetic aperture radar at Venus

Quaternions: $Q = iq_1 + jq_2 + kq_3 + q_4$

$i^2 = j^2 = k^2 = -1$

$ij = k,\ jk = i,\ ki = j$

$ji = -k,\ kj = -i,\ ik = -j$

Equivalent axis: $\hat{e} = e_1\hat{a}_1 + e_2\hat{a}_2 + e_3\hat{a}_3$
$= e_1\hat{b}_1 + e_2\hat{b}_2 + e_3\hat{b}_3$

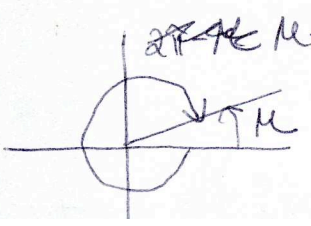
Equivalent angle: μ

$q_1 = e_1 \sin(\mu/2)$ $\quad q_2 = e_2 \sin(\mu/2)$

$q_3 = e_3 \sin(\mu/2)$ $\quad q_4 = \cos(\mu/2)$

$$q = \begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix}$$

$$\|Q\|^2 = q_1^2 + q_2^2 + q_3^2 + q_4^2 = \sin^2\left(\frac{\mu}{2}\right)\underbrace{\left(e_1^2 + e_2^2 + e_3^2\right)}_{=1} + \cos^2\left(\frac{\mu}{2}\right) =$$



$q_1 = e_1 \sin\left(\frac{\mu - 2\pi}{2}\right) = -e_1 \sin(\mu/2)$

$q_2 = -e_2 \sin(\mu/2)$ $\quad q_3 = -e_3 \sin(\mu/3$

Every rotation has two corresponding quaternions:

~~q~~ Q & $-Q$

Find $\hat{e}, \mu$ given q_1, q_2, q_3, q_4

$\mu = 2\cdot\cos^{-1}(q_4)$ $\quad e_i = q_i / \sin(\mu/2)$ $\quad i = 1, 2, 3$

$q_1^2 + q_2^2 + q_3^2 = \sin^2\left(\frac{\mu}{2}\right)(e_1^2 + e_2^2 + e_3^2) = \sin^2\left(\frac{\mu}{2}\right)$

$\sin\left(\frac{\mu}{2}\right) = \pm\sqrt{q_1^2 + q_2^2 + q_3^2}$

$e_i = q_i / \sqrt{q_1^2 + q_2^2 + q_3^2}$ $\qquad \hat{e} = \dfrac{q_1\hat{a}_1 + q_2\hat{a}_2 + q_3\hat{a}_3}{\sqrt{q_1^2 + q_2^2 + q_3^2}}$

$Q \rightarrow$ Rot. Matrix

$$R(q, q_4) = (q_4^2 - q^T q)\mathbb{1} + 2qq^T - 2q_4 S(q)$$

$$= \begin{pmatrix} 1-2(q_2^2+q_3^2) & 2(q_1q_2+q_3q_4) & 2(q_1q_3-q_2q_4) \\ 2(q_1q_2-q_3q_4) & 1-2(q_1^2+q_3^2) & 2(q_2q_3+q_1q_4) \\ 2(q_1q_3+q_2q_4) & 2(q_2q_3-q_1q_4) & 1-2(q_1^2+q_2^2) \end{pmatrix}$$

where $S(q) = \begin{pmatrix} 0 & -q_3 & q_2 \\ q_3 & 0 & -q_1 \\ -q_2 & q_1 & 0 \end{pmatrix}$

No sin/cos $\rightarrow$ faster computation

$R \rightarrow Q$

$R_{11} + R_{22} + R_{33} = 3 - 4(q_1^2 + q_2^2 + q_3^2) = 3 - 4(1 - q_4^2) = -1 + 4q$