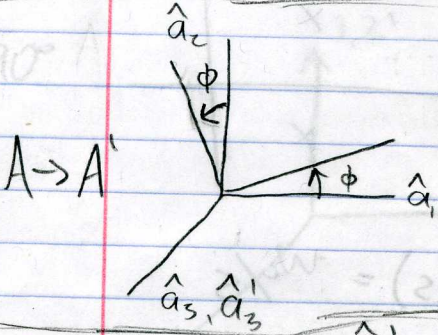


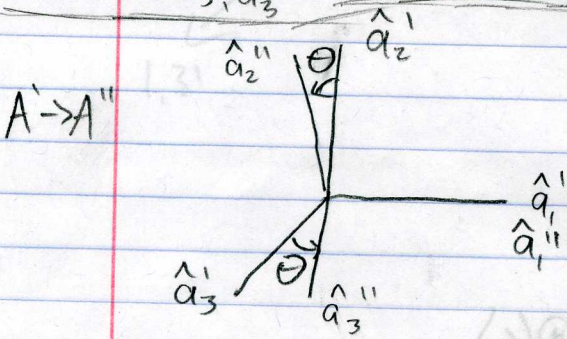
1/31/08

$$A \rightarrow B = \omega_1 \hat{b}_1 + \omega_2 \hat{b}_2 + \omega_3 \hat{b}_3$$

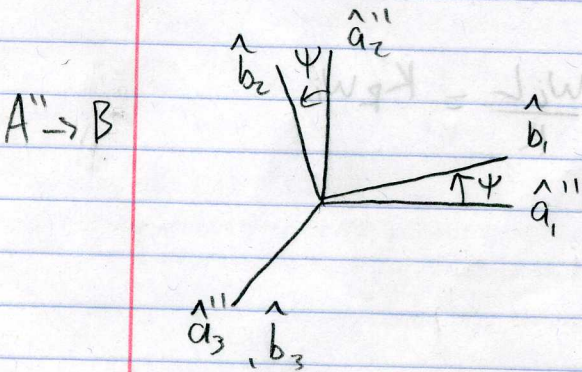
$$\begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix} \neq \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix}$$



$$A \rightarrow A' = \dot{\phi} \hat{a}_3'$$



$$A' \rightarrow A'' = \dot{\theta} \hat{a}_1''$$



$$A'' \rightarrow B = \dot{\psi} \hat{b}_3$$

$$A \rightarrow B = \dot{\phi} \hat{a}_3' + \dot{\theta} \hat{a}_1'' + \dot{\psi} \hat{b}_3$$

$$\begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix} = R_3(\psi) \begin{bmatrix} \dot{\theta} \\ 0 \\ 0 \end{bmatrix}_{A''}$$

$$= \begin{bmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{\theta} \\ 0 \\ 0 \end{bmatrix} \Big|_{A''}$$

$$= \begin{bmatrix} \dot{\theta} \cos\psi \\ -\dot{\theta} \sin\psi \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \\ \\ \dot{\phi} \end{bmatrix} \Big|_B = R_3(\psi) R_1(\theta) \begin{bmatrix} 0 \\ 0 \\ \dot{\phi} \end{bmatrix} \Big|_{A'}$$

$$= \begin{bmatrix} c\psi & s\psi & 0 \\ -s\psi & c\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\theta & s\theta \\ 0 & -s\theta & c\theta \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \dot{\phi} \end{bmatrix} \Big|_{A'}$$

$$= \begin{bmatrix} c\psi & s\psi c\theta & s\psi s\theta \\ -s\psi & c\psi c\theta & c\psi s\theta \\ 0 & -s\theta & c\theta \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \dot{\phi} \end{bmatrix}$$

$$= \begin{bmatrix} \dot{\phi} s\psi s\theta \\ \dot{\phi} c\psi s\theta \\ \dot{\phi} c\theta \end{bmatrix}$$

$$\begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} = \begin{bmatrix} s\psi s\theta \\ c\psi s\theta \\ c\theta \end{bmatrix} \dot{\phi} + \begin{bmatrix} c\psi \\ -s\psi \\ 0 \end{bmatrix} \dot{\theta} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \dot{\psi}$$

$$\begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} = \begin{bmatrix} s\psi s\theta & c\psi & 0 \\ c\psi s\theta & -s\psi & 0 \\ c\theta & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix}$$

↑ (not a rotation matrix)

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \frac{1}{\sin\theta} \begin{bmatrix} s\psi & c\psi & 0 & 0 \\ c\psi s\theta & -s\psi s\theta & 0 & 0 \\ -s\psi c\theta & -c\psi c\theta & s\theta & 0 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix}$$

For $\theta = 0$ (top) or π (bottom)

$$R_{313} = \begin{bmatrix} \cos(\phi \pm \psi) & \sin(\phi \pm \psi) & 0 & 0 \\ \mp \sin(\phi \pm \psi) & \pm \cos(\phi \pm \psi) & 0 & 0 \\ 0 & 0 & 0 & \pm 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ \psi \\ \phi \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ \theta_2 & \theta_1 & 0 \\ \theta_3 & \theta_2 & 0 \end{bmatrix} \begin{bmatrix} 0 & \psi_2 & \psi_1 \\ 0 & \psi_3 & \psi_2 \\ 1 & 0 & 0 \end{bmatrix} =$$

$$\begin{bmatrix} 0 \\ 0 \\ \phi \end{bmatrix} \begin{bmatrix} \theta_2 \psi_2 & \theta_1 \psi_2 & \psi_1 \\ \theta_2 \psi_3 & \theta_1 \psi_3 & \psi_2 \\ \theta_3 & \theta_2 & 0 \end{bmatrix} =$$

$$\begin{bmatrix} \theta_2 \psi_2 \phi \\ \theta_2 \psi_3 \phi \\ \theta_3 \phi \end{bmatrix} =$$

$$\psi \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + \theta \begin{bmatrix} \psi_1 \\ \psi_2 \\ 0 \end{bmatrix} + \phi \begin{bmatrix} \theta_2 \psi_2 \\ \theta_2 \psi_3 \\ \theta_3 \end{bmatrix} = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix}$$

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} \begin{bmatrix} 0 & \psi_1 & \theta_2 \psi_2 \\ 0 & \psi_2 & \theta_2 \psi_3 \\ 1 & 0 & \theta_3 \end{bmatrix} = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix}$$

(xintou naitotou o ton)