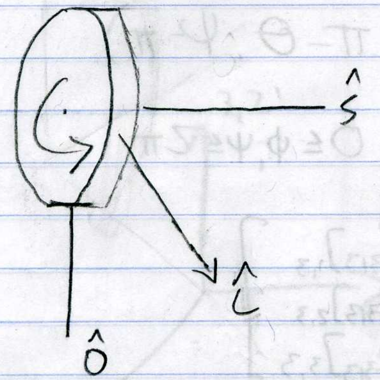
1/29/08

$\theta_1, \theta_2, \theta_3 \longleftrightarrow {}^{A}\vec{w}^{B} = w_1 \hat{b}_1 + w_2 \hat{b}_2 + w_3 \hat{b}_3$

(measured by rate gyros)



$$\sum \text{Torques} = \frac{{}^{A}d\vec{H}}{dt}$$

$$\vec{H} = L\hat{s} + I_i w_i \hat{i} + I_o w_o \hat{o}$$

$L >> 0$

$${}^{A}\vec{w}^{B} = w_i \hat{i} + w_o \hat{o} + w_s \hat{s}$$

$$\frac{{}^{A}d\vec{H}}{dt} = \frac{{}^{B}d\vec{H}}{dt} + {}^{A}\vec{w}^{B} \times \vec{H}$$

$$= \left(\cancelto{0}{\frac{dL}{dt}} \hat{s} + I_i \frac{dw_i}{dt} \hat{i} + I_o \frac{dw_o}{dt} \hat{o}\right) + {}^{A}\vec{w}^{B} \times \vec{H}$$

$$\begin{vmatrix} \hat{i} & \hat{o} & \hat{s} \\ w_i & w_o & w_s \\ I_i w_i & I_o w_o & L \end{vmatrix} = \hat{i}(w_o L - I_o w_s w_o) + \hat{o}(I_i w_s w_i - w_i L) + \hat{s}(I_o w_i w_o - I_i w_i w_o)$$

$\sum$ Torques $=$ (something about $\hat{i}$) $+ \hat{o}(-K\theta_o - Bw_o)$

<u>$\hat{o}$ direction</u>

$$-K\theta_o - Bw_o = I_o \frac{dw_o}{dt} + \cancelto{\approx 0}{I_i w_s w_i} - w_i L$$

≈ 0 (assume small w_o, w_i, w_s)

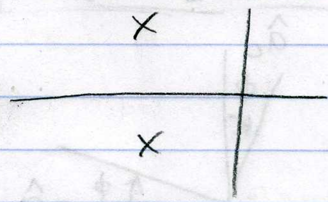
$$I_o \ddot{\theta}_o + B\dot{\theta}_o + K\theta_o = w_i L$$

Laplace Transform

$$I_0 s^2 \Theta(s) + Bs\, \Theta(s) + K\, \Theta(s) = \Omega(s) L$$

$$\frac{\Theta(s)}{\Omega(s)} = \frac{L}{I_0 s^2 + Bs + K}$$

root-locus



$$s = \frac{-B \pm \sqrt{B^2 - 4 I_0 K}}{2 I_0}$$

$$w_i(t) = w_i = \text{const} \quad \rightarrow \Omega(s) = w_i/s$$

$$\Theta(s) = \frac{w_i L}{s(I_0 s^2 + Bs + K)}$$

Final Value Theorem:

$$\theta_{0,ss} = \lim_{t \to \infty} \theta(t) = \lim_{s \to 0} s\, \Theta(s)$$

$$\theta_{0,ss} \lim_{s \to 0} \frac{s w_i L}{\cancel{s}(I_0 s^2 + Bs + K)} = \frac{w_i L}{K} = K_R w_i$$