1/24 Satellite - GPS Block II F

Thrusters, gyros, reaction wheels, star tracker, magnets

3-axis active control

Euler Angle sequence

123	313 ← "classical Euler angle sequence"
312	131
213	232
231	323
132	121
321	212
Asymmetric	Symmetric

$R_{3\,1\,3}(\phi, \theta, \psi) = R_3(\psi)\, R_1(\theta)\, R_3(\phi)$

ϕ: ascending node, θ: inclination (i), ψ: argument of periapse (ω)

$$R_3(\psi) = \begin{bmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_1(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{bmatrix}$$

$$R_3(\phi) = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_{3\,1\,3} = \begin{bmatrix} C\phi C\psi - C\theta S\phi S\psi & C\psi S\phi + C\theta C\phi S\psi & S\theta S\psi \\ -C\theta C\psi S\phi - C\phi S\psi & C\theta C\phi C\psi - S\phi S\psi & C\psi S\theta \\ S\theta S\phi & -C\phi S\theta & C\theta \end{bmatrix}$$

$$R_{3,1,3}(\phi,\theta,\psi) = R_{3,1,3}(\theta+\pi, -\theta, \psi-\pi)$$

constrain the angles: $0 \le \theta \le \pi$

$0 \le \phi < 2\pi$

$0 \le \psi < 2\pi$

$$R_{i,j,k}(\phi,\theta,\psi) = R_{i,j,k}(\phi+\pi, \pi-\theta, \psi-\pi)$$

constrain:

$0 \le \phi \le 2\pi$

$-\frac{\pi}{2} \le \theta < \frac{\pi}{2}$

$0 \le \psi < 2\pi$

$$R_{3,1,3} = \begin{bmatrix} [R_{3,1,3}]_{1,1} & [R_{3,1,3}]_{1,2} & [R_{3,1,3}]_{1,3} \\ [R_{3,1,3}]_{2,1} & [R_{3,1,3}]_{2,2} & [R_{3,1,3}]_{2,3} \\ [R_{3,1,3}]_{3,1} & [R_{3,1,3}]_{3,2} & [R_{3,1,3}]_{3,3} \end{bmatrix}$$

$$\theta = \pm \cos^{-1}\left([R_{3,1,3}]_{3,3}\right)$$

$$\sin\theta\sin\phi = [R_{3,1,3}]_{3,1} \qquad -\sin\theta\cos\phi = [R_{3,1,3}]_{3,2}$$

$$-\tan\phi = \frac{[R_{3,1,3}]_{3,1}/\sin\theta}{[R_{3,1,3}]_{3,2}/\sin\theta} \qquad \tan\theta = \frac{Y}{X},\ \tan(\theta+\pi) = \frac{-Y}{-X}$$

$$\theta = \operatorname{atan2}(Y, X)$$

$$\theta = \cos^{-1}\left([R_{3,1,3}]_{3,3}\right)$$

$$\phi = \operatorname{atan2}\left([R_{3,1,3}]_{3,1}, [R_{3,1,3}]_{3,2}\right)$$

$$\psi = \operatorname{atan2}\left([R_{3,1,3}]_{1,3}, [R_{3,1,3}]_{2,3}\right)$$