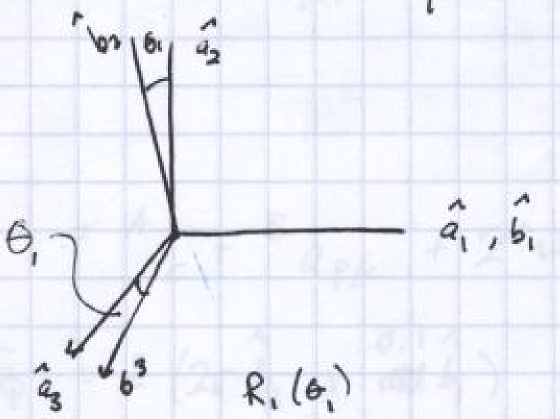
Tuesday 1/22/2007

Satellite of the day - Transit (Precursor to GPS)



$$\begin{bmatrix} \hat{b}_1 \\ \hat{b}_2 \\ \hat{b}_3 \end{bmatrix} = \overbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_1 & \sin\theta_1 \\ 0 & -\sin\theta_1 & \cos\theta_1 \end{bmatrix}}^{R_1(\theta_1)} \begin{bmatrix} \hat{a}_1 \\ \hat{a}_2 \\ \hat{a}_3 \end{bmatrix}$$

$$\begin{bmatrix} X_{B1} \\ X_{B2} \\ X_{B3} \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_1 & \sin\theta_1 \\ 0 & -\sin\theta_1 & \cos\theta_1 \end{bmatrix}}_{\substack{R_1(\theta_1) \\ R^{B/A}}} \begin{bmatrix} X_{A_1} \\ X_{A_2} \\ X_{A_3} \end{bmatrix}$$

orthogonal $\leftarrow$ $\vec{r}_1 \cdot \vec{r}_2 = 0$

$c_1^T c_2 = 0$

$$R_1(-\theta_1) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_1 & -\sin\theta_1 \\ 0 & \sin\theta_1 & \cos\theta_1 \end{bmatrix} = R_1(\theta_1)^T$$

$$\cancel{R(\theta_1)^T R(\theta_1) =} \begin{bmatrix} \hat{a}_1 \\ \hat{a}_2 \\ \hat{a}_3 \end{bmatrix} = R_1(-\theta_1) R_1(\theta_1) \begin{bmatrix} \hat{a}_1 \\ \hat{a}_2 \\ \hat{a}_3 \end{bmatrix}$$

$$R_1(\theta_1)^T R_1(\theta_1) =$$

$$R_1(\theta_1)^T R_1(\theta_1) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_1 & -s_1 \\ 0 & s_1 & c_1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_1 & s_1 \\ 0 & -s_1 & c_1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_1(\theta_1) R_1(\theta_1)^T = I$$

$$R_1(\theta_1)^{-1} = R_1(\theta_1)^T$$

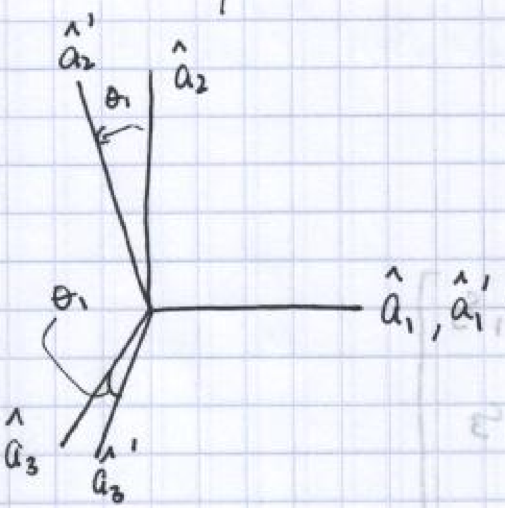
Key Properties

$$\underbrace{\begin{array}{l} R^{-1} = R^T \\ \det R = +1 \end{array}}_{\text{special orthogonal}}$$

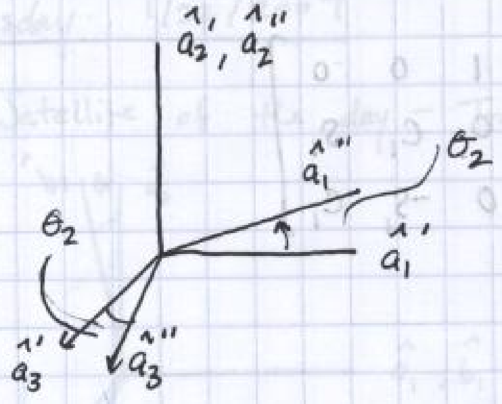
$$\det R_1(-\theta_1) = (1(c_1)(c_1) + 0 + 0) - (1(-s_1)(s_1) + 0 + 0) = 1$$

$R \in SO(3)$

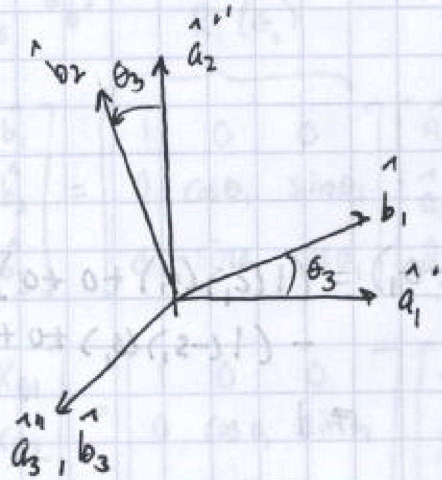
say $R_1, R_2 \in SO(3)$ then $R_1 R_2 \in SO(3)$



$$\begin{bmatrix} \hat{a}_1' \\ \hat{a}_2' \\ \hat{a}_3' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_1 & s_1 \\ 0 & -s_1 & c_1 \end{bmatrix}$$



$$\begin{bmatrix} \hat{a}_1'' \\ \hat{a}_2'' \\ \hat{a}_3'' \end{bmatrix} = \overbrace{\begin{bmatrix} \cos\theta_2 & 0 & -\sin\theta_2 \\ 0 & 1 & 0 \\ \sin\theta_2 & 0 & \cos\theta_2 \end{bmatrix}}^{R_2(\theta_2)} \begin{bmatrix} \hat{a}_1' \\ \hat{a}_2' \\ \hat{a}_3' \end{bmatrix}$$



$$\begin{bmatrix} \hat{b}_1 \\ \hat{b}_2 \\ \hat{b}_3 \end{bmatrix} = \begin{bmatrix} \cos\theta_3 & \sin\theta_3 & 0 \\ -\sin\theta_3 & \cos\theta_3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \hat{a}_1'' \\ \hat{a}_2'' \\ \hat{a}_3'' \end{bmatrix}$$

$$\begin{bmatrix} \hat{b}_1 \\ \hat{b}_2 \\ \hat{b}_3 \end{bmatrix} = \underbrace{R_3(\theta_3)\, R_2(\theta_2)\, R_1(\theta_1)}_{R_{123}(\theta_1, \theta_2, \theta_3)} \begin{bmatrix} \hat{a}_1 \\ \hat{a}_2 \\ \hat{a}_3 \end{bmatrix}$$

$$R_{123} = \begin{bmatrix} c_2 c_3 & s_1 s_2 c_3 + c_1 s_3 & -c_1 s_2 c_3 + s_1 s_3 \\ -c_2 s_3 & -s_1 s_2 s_3 + c_1 c_3 & c_1 s_2 s_3 + s_1 c_3 \\ s_2 & -s_1 c_2 & c_1 c_2 \end{bmatrix}$$

Example $\theta_1 = 90°$, $\theta_2 = 0°$ $\theta_3 = 90°$

A vector has coordinates $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ in A. What are the coords in B?

Ans: $R_{123} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}$

$$R_{123} = \begin{bmatrix} 0 & 0 & 1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix}$$