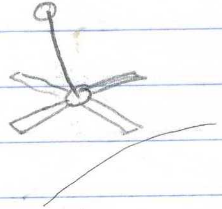
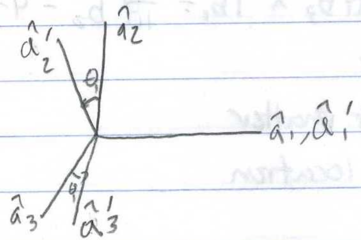
1/22/08 Satellite of the Day: Transit

- gravity gradient stabalization - passive method



- 1960s craft
- precursor to GPS, first satellite navigation system
- used to correct inertia measurements in submarines
- Hawaii mis-mapped by 1km
- accuracy to ~25m
- 1100 km orbit



$$\begin{bmatrix} \hat{a}_1' \\ \hat{a}_2' \\ \hat{a}_3' \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_1 & \sin\theta_1 \\ 0 & -\sin\theta_1 & \cos\theta_1 \end{bmatrix}}_{R_1(\theta_1)} \begin{bmatrix} \hat{a}_1 \\ \hat{a}_2 \\ \hat{a}_3 \end{bmatrix}$$

$$\begin{bmatrix} x_{B1} \\ x_{B2} \\ x_{B3} \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_1 & \sin\theta_1 \\ 0 & -\sin\theta_1 & \cos\theta_1 \end{bmatrix}}_{R_1(\theta_1) \leftarrow R^{B/A}} \begin{bmatrix} x_{A1} \\ x_{A2} \\ x_{A3} \end{bmatrix}$$

$R_1(\theta_1) \leftarrow R^{B/A}$ (rotation matrix 'from A to B')

orthogonal: $\vec{r}_1 \circ \vec{r}_2 = 0$

$$R_1(-\theta_1) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_1 & -\sin\theta_1 \\ 0 & \sin\theta_1 & \cos\theta_1 \end{bmatrix} \Rightarrow R_1(\theta_1)^T = R_1(-\theta_1)$$

$$\begin{bmatrix} \hat{a}_1 \\ \hat{a}_2 \\ \hat{a}_3 \end{bmatrix} = \underbrace{R_1(-\theta_1)\, R_1(\theta_1)}_{I} \begin{bmatrix} \hat{a}_1 \\ \hat{a}_2 \\ \hat{a}_3 \end{bmatrix}$$

$$R_1(\theta_1)^T R_1(\theta_1) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_1 & -\sin\theta_1 \\ 0 & \sin\theta_1 & \cos\theta_1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_1 & \sin\theta_1 \\ 0 & -\sin\theta_1 & \cos\theta_1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

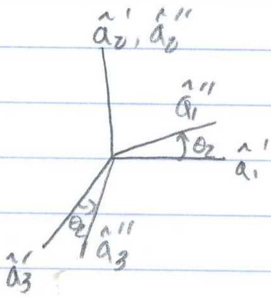
$\Rightarrow R_1(\theta_1)\, R_1(\theta_1)^T = I$

↳ $R_1(\theta_1)^{-1} = R_1(\theta_1)^T$

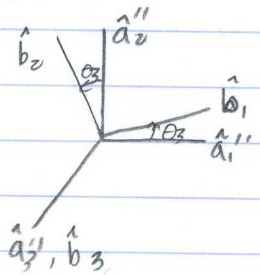
- all rotational matrices are orthonormal ⇒ transpose = inverse ⇒ $R^{-1} = R^T$
- if use rotation matrix R to go from frame A to B, just use transpose of R to go from B back to A
- for a rotation matrix: $\underbrace{\det R = +1}_{\text{special orthogonal}}$ ⇒ $R \in SO(3)$ ← 3×3 matrices (↳ special orthogonal)

$R_1, R_2 \in SO(3)$

↳ $R_1 R_2 \in SO(3)$



$$\begin{bmatrix} \hat{a}_1'' \\ \hat{a}_2'' \\ \hat{a}_3'' \end{bmatrix} = \underbrace{\begin{bmatrix} \cos\theta_2 & 0 & -\sin\theta_2 \\ 0 & 1 & 0 \\ \sin\theta_2 & 0 & \cos\theta_2 \end{bmatrix}}_{R_2(\theta_2)} \begin{bmatrix} \hat{a}_1' \\ \hat{a}_2' \\ \hat{a}_3' \end{bmatrix}$$



$$\begin{bmatrix} \hat{b}_1 \\ \hat{b}_2 \\ \hat{b}_3 \end{bmatrix} = \underbrace{\begin{bmatrix} \cos\theta_3 & \sin\theta_3 & 0 \\ -\sin\theta_3 & \cos\theta_3 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{R_3(\theta_3)} \begin{bmatrix} \hat{a}_1'' \\ \hat{a}_2'' \\ \hat{a}_3'' \end{bmatrix}$$

$$\begin{bmatrix} \hat{b}_1 \\ \hat{b}_2 \\ \hat{b}_3 \end{bmatrix} = \underbrace{R_3(\theta_3)\, R_2(\theta_2)\, R_1(\theta_1)}_{R_{123}(\theta_1, \theta_2, \theta_3)} \begin{bmatrix} \hat{a}_1 \\ \hat{a}_2 \\ \hat{a}_3 \end{bmatrix}$$

$$R_{123}(\theta_1, \theta_2, \theta_3) = \begin{bmatrix} C_2 C_3 & S_1 S_2 C_3 + C_1 S_3 & -C_1 S_2 C_3 + S_1 S_3 \\ -C_2 S_3 & -S_1 S_2 S_3 + C_1 C_3 & C_1 S_2 S_3 + S_1 C_3 \\ S_2 & -S_1 C_2 & C_1 C_2 \end{bmatrix}$$

$R_{123}^{-1} = R_{123}^T$ ✓

$\det R_{123} = +1$ ✓

ex $\theta_1 = 90^\circ$, $\theta_2 = 0^\circ$, $\theta_3 = 90^\circ$ A vector has coordinates $\begin{bmatrix}1\\1\\1\end{bmatrix}$ in A.

What are coordinates in B?

$$R_{123}\begin{bmatrix}1\\1\\1\end{bmatrix} = \begin{bmatrix}0 & 0 & 1\\-1 & 0 & 0\\0 & -1 & 0\end{bmatrix}\begin{bmatrix}1\\1\\1\end{bmatrix} = \begin{bmatrix}1\\-1\\-1\end{bmatrix}$$