AE 252: Introduction to Aerospace Dynamics
Spring 2007

Homework #7

Due Thursday, March 15, at 5PM in the "AE252" box in the mailroom. Please start every problem on a new sheet of paper, and put your name on the *back* of each page.

<u>NOTE: The third problem *will* be graded and will be worth 5 points. One of the first two problems will also be graded and will be worth 3 points. You will receive 2 points if you do the fourth problem.</u>

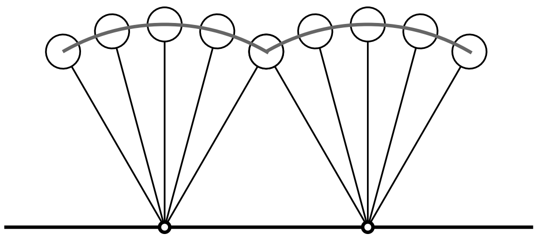
1. Text problem 16.47 (don't forget to draw coordinate axes).

2. Text problem 16.109 (use the rocket equation as derived in class).

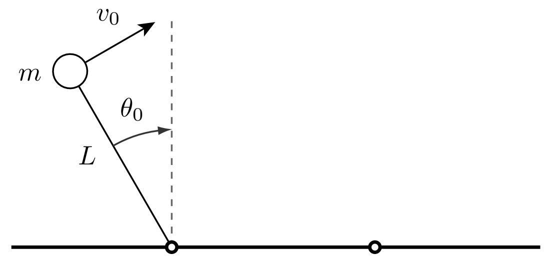
3. Do the "simple model of walking" problem on the following page.

4. Pose a problem of your own. (A homework or exam problem, a design problem, a "real-world" problem, …) Then, suggest a solution.

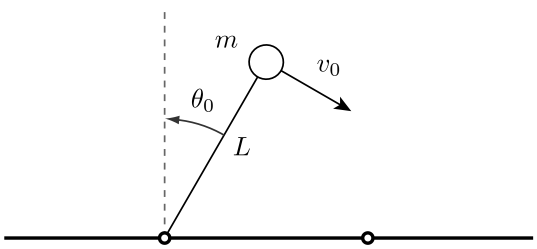
Problem 3. A "simple" model of walking.



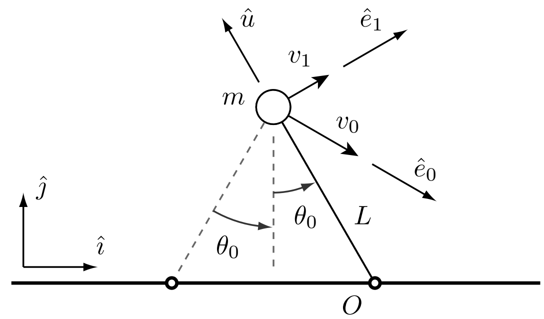
(a) Walking with continuous contact (no jumping)



(b) At the beginning of a step



(c) At the end of a step



(d) At impact (placing the front leg, lifting the rear leg)

Last week you looked at gibbon locomotion. This week you will look at the "upside-down" version of that problem—namely, *walking*. Since we did not have time to work out the details in class, you get the chance to do it yourself!

Model the walker as a particle of mass $m = 50$ kg with massless legs of length $L = 1.0$ m. It moves in a vertical plane where gravity acts downward and $g = 9.8$ m/s^2. It begins each step with speed v_0. Then, it swings through an arc of angle $2\theta_0$ with a leg fixed on the ground—at this point, its speed is again v_0. Finally, it ends each step by placing a new leg on the ground.

Each time a new leg is placed on the ground, there is a collision (see Fig. 1(d)). We model this collision as plastic. It "instantaneously" changes the velocity vector from $\vec{v}_0 = v_0\hat{e}_0$ to some $\vec{v}_1 = v_1\hat{e}_1$. As we discussed in class, angular momentum about point O is conserved during this collision.

a. Using conservation of angular momentum, find the speed v_1 after collision in terms of v_0 and θ_0. *(Hint: notice that v_0 must be perpendicular to the rear leg, and v_1 must be perpendicular to the front leg. I've drawn some coordinate vectors that may help. Apply the conservation principle in cartesian coordinates, by expressing $\hat{u}$, $\hat{e}_0$, and $\hat{e}_1$ in terms of $\hat{\imath}$ and $\hat{\jmath}$.)*

b. In part (a) you should have found that $v_1 \leq v_0$. So with every step, the walker slows down. Assuming $\theta_0 = 15°$ (typical for a person), find the total number of steps k that can be taken before stopping, as a function of the initial speed v_0. Use MATLAB to plot your results. Does your answer depend on the mass m? *(Hint: first apply the principle of work and energy to find the minimum initial speed v_{min} necessary to take a single step. Walking stops when the speed after impact is less than v_{min}.)*

Let's say you want to maintain a constant speed while walking, so $v_1 = v_0$ after every step. There are two ways you might think about doing this. First, you could push off with the back leg just before placing the front leg on the ground (just *before* impact). Second, you could apply a torque about point O with the front leg just after placing it on the ground (just *after* impact). You can compare these two approaches in terms of the work required—in other words, in terms of the change in kinetic energy.

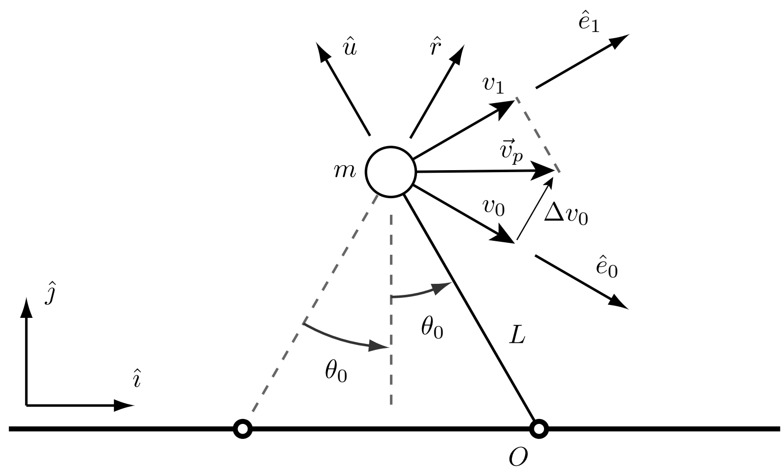
c. *(Impulse before impact.)* Model "push-off" as an impulse $p\hat{r} = \int_t^{t+\Delta t} F\hat{r}dt$ exerted over a short time Δt by the rear leg, just before impact.

   i. Apply the principle of linear impulse and momentum to compute the velocity $\vec{v}_p$ just before impact, in terms of p. (See the picture below.)

   ii. Apply conservation of angular momentum about point O to find the speed v_1 just after impact, in terms of p.

   iii. Find the impulse p required so that $v_1 = v_0$.

   iv. Assuming $v_0 = 1$ m/s, find the total work W_{before} done by the impulse as a function of θ_0. *(Hint: the work done by the impulse is equal to the change in kinetic energy that it causes. Note that this happens before the front leg hits the ground.)*



d. *(Impulse after impact.)* In part (a) you found v_1 in terms of v_0 and θ_0, assuming that no impulse was applied before impact. Model "front-leg-torque" as an angular impulse that occurs just after impact. Assuming $v_0 = 1$ m/s, find the total work W_{after} done by this angular impulse as a function of θ_0 if it quickly accelerates the mass from v_1 back to the original speed v_0. *(Hint: you don't need to worry about the details of the impulse as you did in part (c)—just compute the change in kinetic energy.)*

e. Finally, plot the ratio $W_{\text{after}}/W_{\text{before}}$. For a typical stride angle $\theta_0 = 15°$, which strategy is more efficient? For what angle θ_0 would your answer change? (Does this make sense?)

f. **BONUS (2 pts):** Imagine the ground is sloped downward at an angle ϕ. For what ϕ does the walker continue at a constant speed $v_1 = v_0$ after each step, without any additional impulse as in parts (c)-(e)? *(This is called "passive" walking. In fact, it is fairly easy to build a passive walker out of something like legos or tinkertoys. You can get an* **extra 2 pts bonus***—for a total of 4—if you build a passive walker yourself, and demonstrate it during class.)*

(For more details, see Kuo, "Energetics of Actively Powered Locomotion Using the Simplest Walking Model," *J. Biomech. Engr.*, 124:113-120, Feb 2002.)