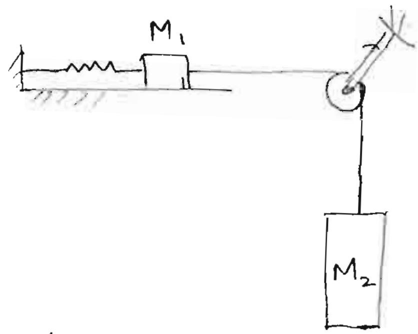
PROB 15.57




Initial extension in spring:

$$k(x) = 50\text{ N}$$

$$\Rightarrow x = \frac{50\text{ N}}{100\text{ N/m}} = 0.5\text{ m}$$

NEWTON'S LAWS:

Now overall external forces acting on system (along string)

① $M_2 g$

② Friction $= -\mu N_1 = -\mu m_1 g$

③ Spring $= -k(x) = -kx$

∴ Acceleration along string (which will be same for both masses)

$$\boxed{\ddot{x} = \frac{M_2 g - \mu M_1 g - kx}{M_2 + M_1}}$$

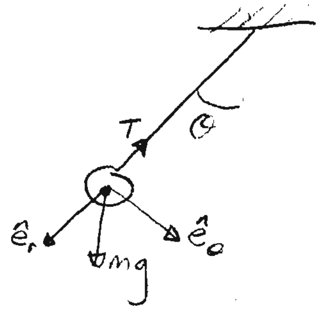
$$\frac{v\,dv}{dx} = 5.43 - 4.16x$$

$$\Rightarrow \frac{v^2}{2} - 0 = \int_{0.5}^{1.5} (5.43 - 4.16x)\,dx$$

$$\Rightarrow \frac{v^2}{2} = 3.35$$

$$\Rightarrow v = \sqrt{6.7} = \boxed{2.59\text{ m/s}}$$

$\boxed{3}$ Use pendulum model




$$\sum F_r = -T + mg\cos\theta$$

$$\sum F_\theta = mg\sin\theta$$

$$\bar{a} = (\ddot{r} - r\dot{\theta}^2)\hat{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{e}_\theta$$

$$\dot{r} = \ddot{r} = 0$$

$$\left.\begin{array}{l} a_r = -r\dot{\theta}^2 \\ a_\theta = r\ddot{\theta} \end{array}\right\} \text{and } r = L$$

Newton

$$\begin{array}{l} \sum F_r = ma_r \\ \sum F_\theta = ma_\theta \end{array} \Longrightarrow \begin{array}{l} mg\cos\theta - T = -mL\dot{\theta}^2 \\ mg\sin\theta = mL\ddot{\theta} \end{array}$$

EOM

$$T = mg\cos\theta + mL\dot{\theta}^2 \quad (1)$$

$$\ddot{\theta} = \frac{g}{L}\sin\theta \quad (2)$$

To find max tension, we need to know where (which $\theta$) $T_m$ occurs. Differentiate $T$ w.r.t. $\theta$:

$$\frac{dT}{d\theta} = -mg\sin\theta + 2mL\dot{\theta}\frac{d\dot{\theta}}{d\theta} \quad (3)$$

To find $\frac{d\dot{\theta}}{d\theta}$, use eqn ②

$$\ddot{\theta} = \frac{g}{L} \sin\theta$$

$$\Rightarrow L\dot{\theta}\frac{d\dot{\theta}}{d\theta} = g\sin\theta$$

Plug this into eqn ③

$$\frac{dT}{d\theta} = -mg\sin\theta + 2m(g\sin\theta)$$

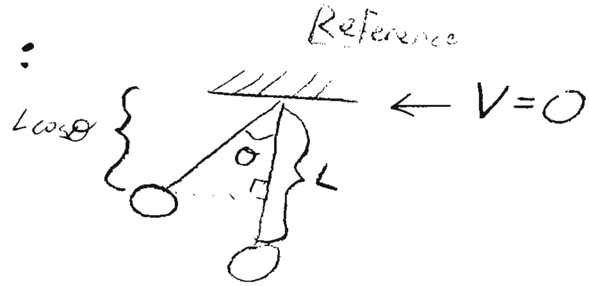
Setting $\frac{dT}{d\theta} = 0$ to find maxima, we find that

$$\theta_m = 0$$

So max tension is at the bottom of the trajectory.

We want $T_m = mg\cos\theta_m + mL\dot{\theta}_m^2$ ④

Lets use energy to find $\dot{\theta}_m$:




$$E_0 = E_m$$

$$T_0 + V_0 = T_m + V_m$$

$$\frac{1}{2}m(L\dot{\theta}_0)^2 + (-mgL\cos\theta_0) = \frac{1}{2}m(L\dot{\theta}_m)^2 + (-mgL)$$

$$\Rightarrow mL\dot{\theta}_m^2 = 2mg(1-\cos\theta_0) + mL^2\dot{\theta}_0^2 \quad ⑤$$

For continuous contact, $\dot{\theta}_0 = 0$

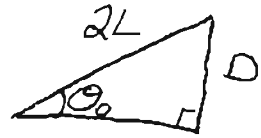
$$mL\dot{\theta}_m^2 = 2mg(1-\cos\theta_0)$$

Plugging this into eqn ④:

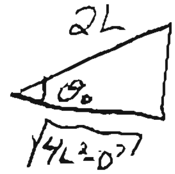
$$T_m = mg\cos\theta_m + 2mg(1-\cos\theta_0) = 3mg - 2mg\cos\theta_0$$

To find $\theta_0$:

We know $\sin\theta_0 = \dfrac{D}{2L} \Rightarrow$




Using pythagarus $\Rightarrow$

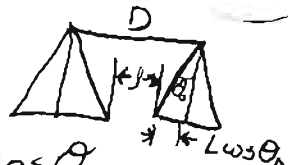



$\Rightarrow \cos\theta_0 = \dfrac{\sqrt{4L^2-D^2}}{2L}$

So, for continuous contact,

$$\boxed{T_m = 3mg - 2mg\,\frac{\sqrt{4L^2-D^2}}{2L}}$$

Intermittent Contact (IC)




The distance traveled w/o rope $\equiv \ell = D - 2L\cos\theta_0$

It leaves the rope w/ $\vec{v}_0 = v_0\cos\theta_0\,\hat{x} + v_0\sin\theta_0\,\hat{y}$

Kinematically $v_0\cos\theta_0 = \dfrac{\ell}{t_f}$ where $t_f$ is the time of freefall

$$\Rightarrow t_f = \frac{\ell}{v_0\cos\theta_0} \quad ⑥$$

In the $\hat{y}$ direction: $y = y_0 + v_0\sin\theta_0\, t - \frac{1}{2}gt^2$

And, $y - y_0 = 0$ IF $t = t_f$

Now,

$$v_0 \sin\theta_0 \, t_f = \frac{1}{2} g t_f^2$$

w/ a non-trivial solution of $t_f = \dfrac{2 v_0 \sin\theta_0}{g}$ (7)

Using (6) and (7)

$$\frac{2 v_0 \sin\theta_0}{g} = \frac{l}{v_0 \cos\theta_0}$$

Or,

$$v_0^2 = \frac{g(D - 2L\cos\theta_0)}{2\sin\theta_0 \cos\theta_0}$$

and $$\dot{\theta}_0^2 = \frac{g(D - 2L\cos\theta_0)}{2L^2 \sin\theta_0 \cos\theta_0} = \frac{g}{L^2}(D - \sqrt{2}L) \quad \text{If } \theta_0 = 45$$

Using the energy eqn (5) we find the new $\dot{\theta}_m$ for IC

$$L\dot{\theta}_m^2 = 2g\left(1 - \frac{\sqrt{2}}{2}\right) + \frac{g}{L}(D - \sqrt{2}L) = \frac{gD}{L} - 2g(1 - \sqrt{2})$$

Using this and eqn (4)

$$T_m = mg + mg\left[\frac{D}{L} + 2(\sqrt{2} - 1)\right]$$ Intermittent contact

$$T_m = mg\left(3 - 2\sqrt{2} + \frac{D}{L}\right)$$

From the plots, it is clear that Intermittent contact should be used for $D = \sqrt{2}L, \frac{3L}{2}, 2L, 4L$

To plot:

Continuous contact: parameter

$$\theta = \text{linspace}\left(\frac{3\pi}{2} - \theta_0, \frac{3\pi}{2} + \theta_0, \#\right)$$

$x_1 = L\cos\theta$

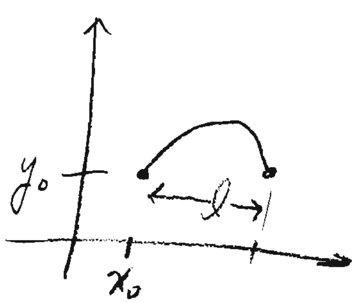
$y_1 = L\sin\theta$ plots first circular arc

$x_2 = x_1 + D$

$y_2 = y_1$ plots second arc

Intermittent: Circular parts are the same

Parabolic trajectory: parameter = time, $x$




$$x = \text{linspace}(x_0, x_0 + l, \#)$$

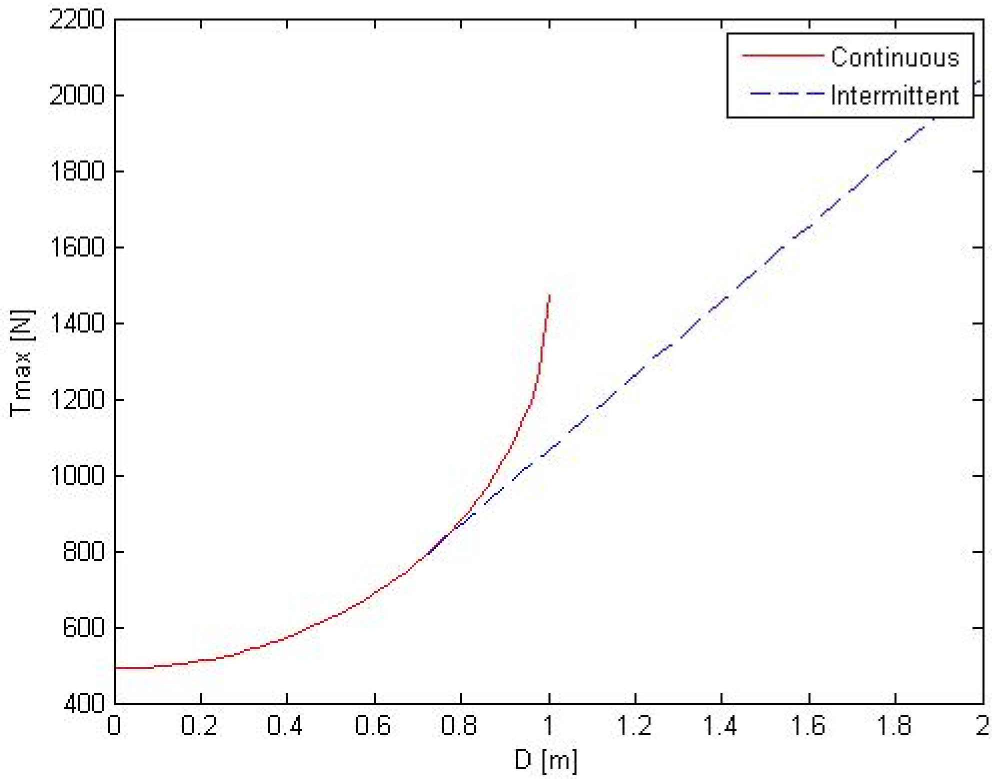
$$y = y_0 + v_0\sin\theta_0 t - \frac{1}{2}gt^2$$

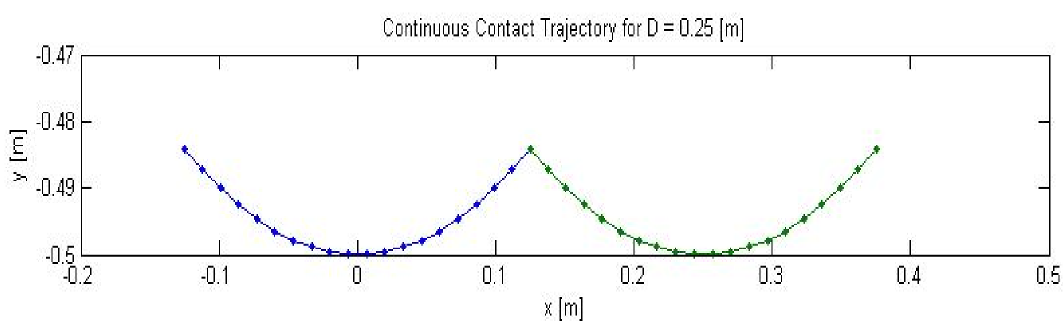
and $t = \dfrac{x - x_0}{v_0\cos\theta_0}$

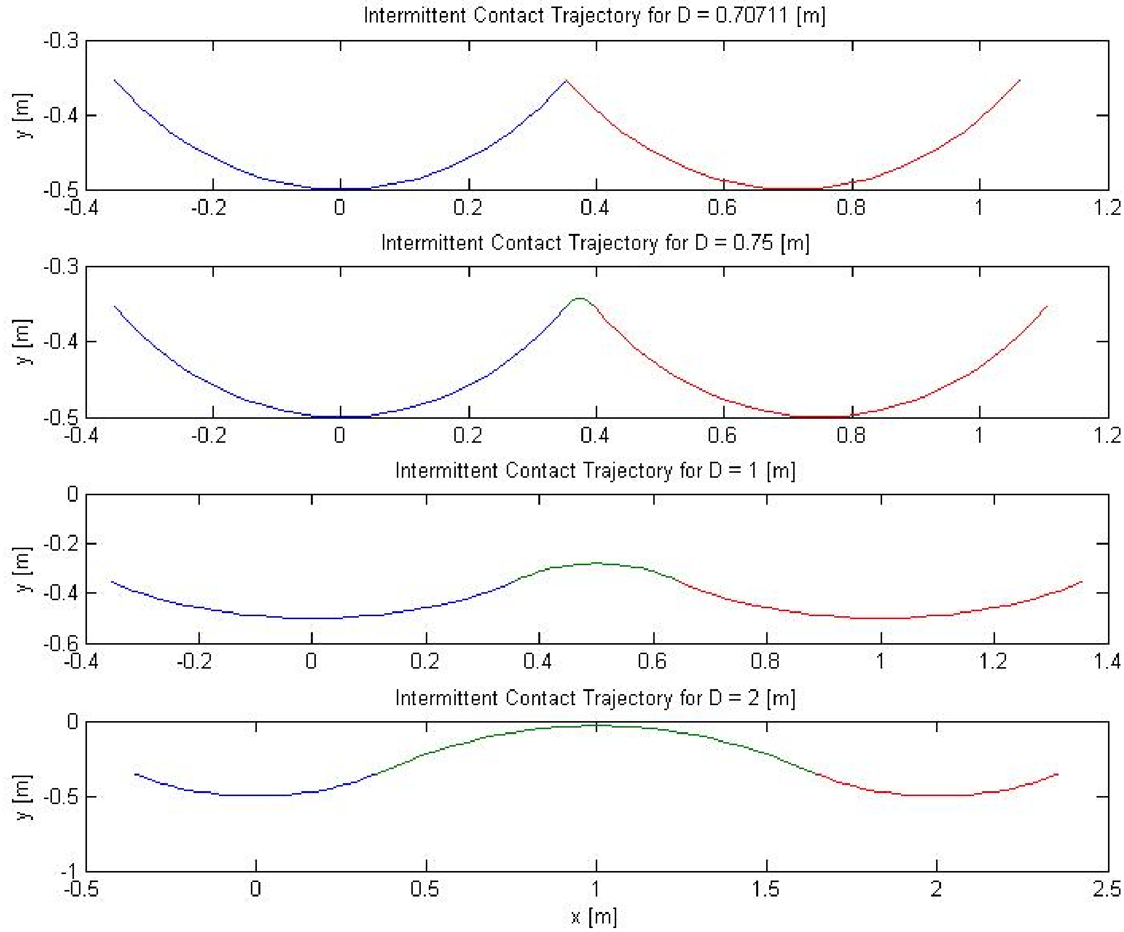
```
% Constants
m=50;
L=0.5;
g=9.81;

% Continuous Contact (cc)
Dcc=linspace(0,2*L,50);
Tmaxcc=-2*m.*g.*sqrt(4*L^2-Dcc.^2)./2./L+3*m*g;

% Intermittant Contact (ic)
Dic=linspace(L*sqrt(2),4*L,50);
Tmaxic=m*g*(3-2*sqrt(2)+Dic./L);

% Plot part c
figure(1)
plot(Dcc,Tmaxcc,'-r',Dic,Tmaxic,'--b')
xlabel('D [m]')
ylabel('Tmax [N]')
legend('Continuous','Intermittent')



% Continuous Contact Trajectory
Dvec=[L/2];
Dsize=size(Dvec);
for i=1:Dsize(2)  %loop in case there are multiple plots
    theta0=asin(Dvec(i)/2/L);
    theta1=linspace(-theta0,theta0,20); %parameter for plotting an arc
    x1=L*cos(theta1-pi/2);
    y1=L*sin(theta1-pi/2);
    figure(2)
    subplot(Dsize(2),1,i)
    plot(x1,y1,'.-',x1+Dvec(i),y1,'.-')
    title(['Continuous Contact Trajectory for D = ', num2str(Dvec(i)),
' [m]'])
    ylabel('y [m]')
end

xlabel('x [m]')

% Intermittent Contact Trajectories
figure(3)
clf
hold on
Dvec=[L*sqrt(2) 3*L/2 2*L 4*L];
Dsize=size(Dvec)
for i=1:Dsize(2)   %loop in case there are multiple plots
    theta=linspace(-pi/4,pi/4,30);
    v0=sqrt(g*(Dvec(i)-sqrt(2)*L));
    x1=L*cos(theta-pi/2);
    y1=L*sin(theta-pi/2);
    x2=linspace(L/sqrt(2),L/sqrt(2)+Dvec(i)-sqrt(2)*L,30);
    y2=y1(1)+x2-L/sqrt(2)-g*((x2-L/sqrt(2))/v0).^2;
    subplot(Dsize(2),1,i)
    plot(x1,y1,x2,y2,x1+Dvec(i),y1)
```

```
    title(['Intermittent Contact Trajectory for D = ',
num2str(Dvec(i)), ' [m]'])
    ylabel('y [m]')
end
xlabel('x [m]')
```

# OUTPUT




Intermittent Contact Trajectory for D = 0.70711 [m]
Intermittent Contact Trajectory for D = 0.75 [m]
Intermittent Contact Trajectory for D = 1 [m]
Intermittent Contact Trajectory for D = 2 [m]
y [m]
x [m]