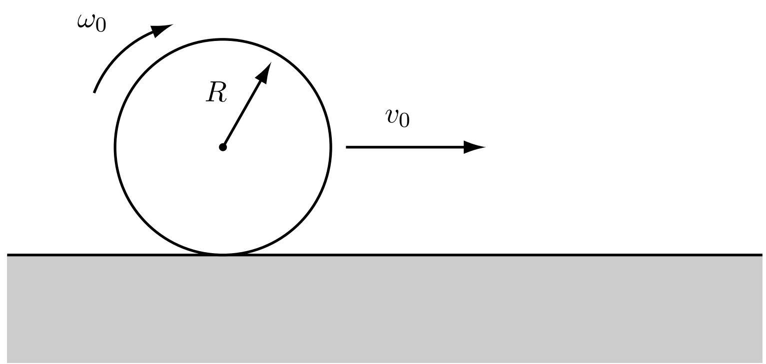
AE 252 Final Exam (Spring 2007)

T. Bretl

NOTE: Please do all four problems. They are each worth 25%.

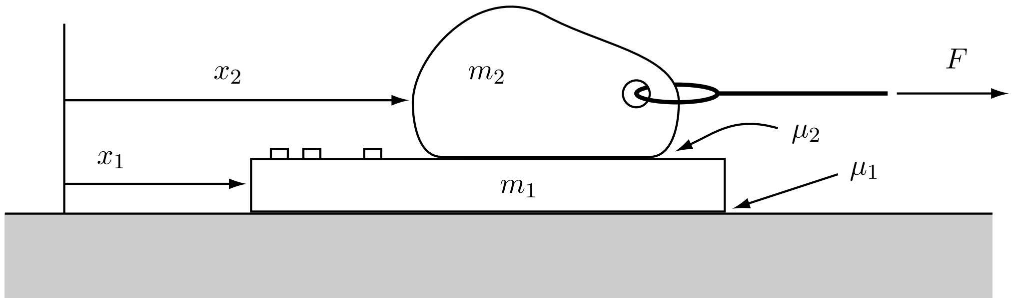


Problem 1 (bowling). To celebrate finishing this exam, you decide to go bowling. You notice that when you throw the ball, it initially slips but eventually begins to roll without slipping. Not being able to get this class out of your mind, you try to understand this phenomenon.

Model the bowling ball as a uniform sphere with mass $m = 7$ kg and radius $R = 150$ mm. It is released with an initial velocity $v_0 = 6$ m/s and an initial angular velocity $\omega_0 = 0$. The coefficient of friction between the ball and the floor is $\mu = 0.1$. Gravity acts downward and $g = 9.81\ \text{m/s}^2$.

a. (10 pts) Write the equations of motion for the ball. *(Remember it is a sphere!)*

b. (15 pts) Find the time at which the ball begins to roll without slipping. *(Hint: what will be the relationship between v and ω at this time?)*

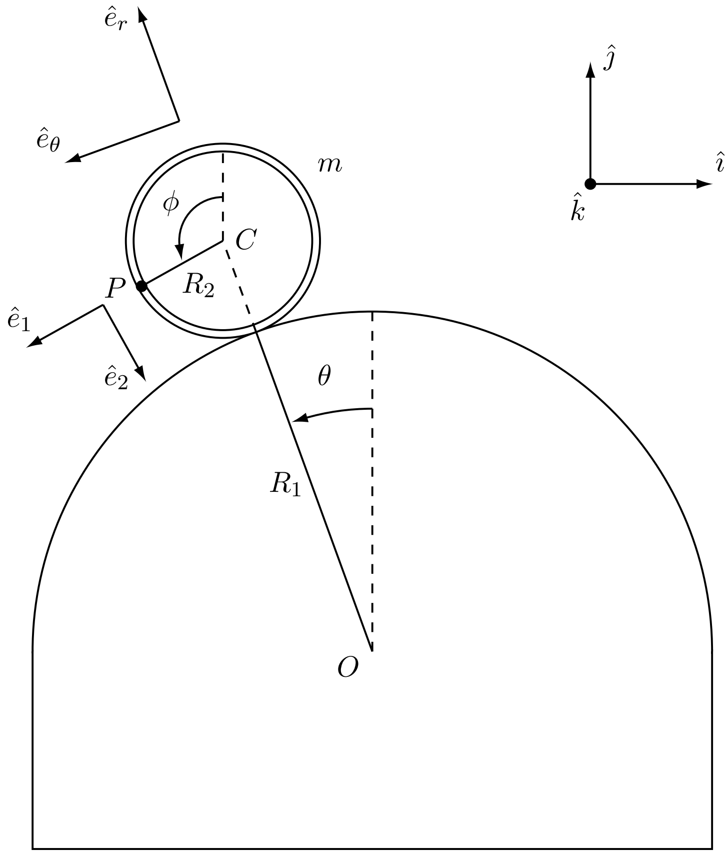


Problem 2 (escape). Feeling trapped, you start daydreaming about escaping from prison. Luckily a guard has dropped the key to your cell on the floor. You tie your shoelace to an old piece of gum and manage to throw the gum onto the key. Since the gum is stickier than the floor, you hope to drag the key toward you by pulling on the shoelace.

The key has mass $m_1 = 0.1$ kg and the gum has mass $m_2 = 0.02$ kg. The coefficient of friction between the floor and the key is $\mu_1 = 0.1$ and the coefficient of friction between the key and the gum is $\mu_2 = 1.0$. Assume neither the key nor the gum rotates. Gravity acts downward and $g = 9.81$ m/s^2.

a. (10 pts) Draw two free-body diagrams, one for the key and one for the gum. Write the equations of motion for each object.

b. (15 pts) Find the maximum force F that can be applied to the shoelace so that the key slips on the floor but the gum does not slip on the key. Also find the resulting acceleration. *(Hint: what is the relationship between $\ddot{x}_1$ and $\ddot{x}_2$ when the gum does not slip?)*



Problem 3 (déjà vu). A hoop of radius $R_2 = 0.5$ m and mass $m = 20$ kg rolls without slipping on a fixed semicircular surface of radius $R_1 = 5$ m. Assume the hoop starts from rest at $\theta_0 = \dot{\theta}_0 = 0$ and $\phi_0 = \dot{\phi}_0 = 0$, and is given a slight nudge to the left. Gravity acts downward (in the $-\hat{\jmath}$ direction) and $g = 9.81$ m/s^2.

a. (5 pts) Write ${}^{\mathcal{F}}\vec{v}_P$ in terms of $\dot{\theta}$, $\dot{\phi}$, R_1, and R_2. You should be able to express your answer using only the coordinate vectors $\hat{e}_\theta$ and $\hat{e}_2$. *(Hint: what is the relationship between the velocity of two points on a rigid body?)*

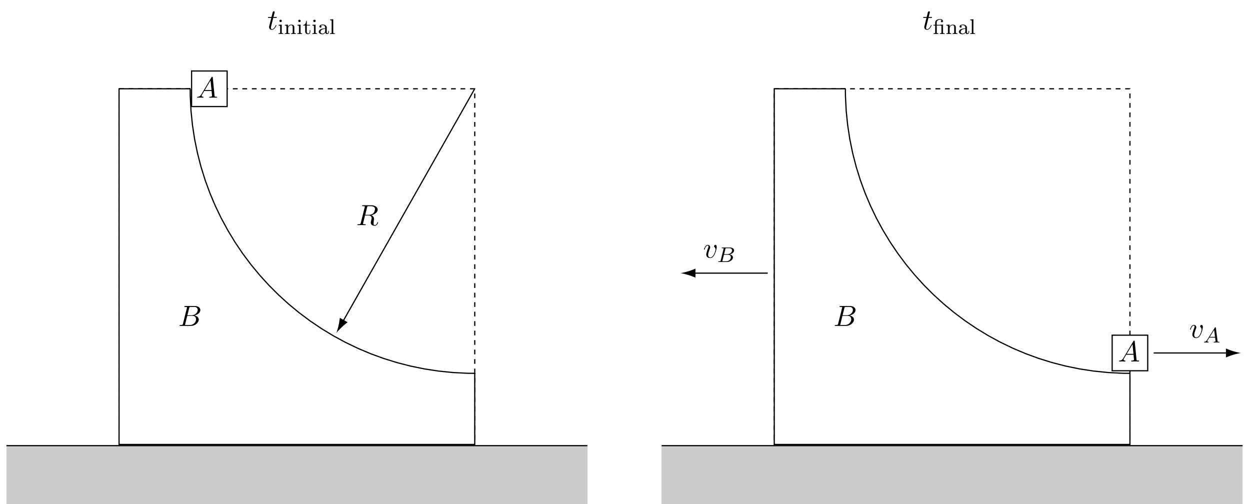
b. (5 pts) Find the relationship between $\dot{\theta}$ and $\dot{\phi}$ using your answer from part (a). *(Hint: when P is in contact with the semicircle, $\hat{e}_2 = -\hat{e}_\theta$. What is ${}^{\mathcal{F}}\vec{v}_P$ at that instant?)*

c. (3 pts) Find the acceleration ${}^{\mathcal{F}}\vec{a}_C$ and angular acceleration ${}^{\mathcal{F}}\vec{\alpha}^B$ of the hoop.

d. (3 pts) Draw a free-body diagram for the hoop and find both $\sum \vec{F}$ and $\sum \vec{M}_C$.

e. (3 pts) Find the equations of motion for the hoop in terms of θ. *(If you could not solve part (b), assume $\dot{\phi} = 5\dot{\theta}$.)*

f. (6 pts) Integrate the equations of motion to find the angle at which the hoop loses contact with the surface.



Problem 4 (slide). A small block of mass $m_A = 1$ kg is dropped from the position shown at left, and slides down a circular path of radius $R = 5$ m cut into a large block of mass $m_B = 10$ kg. The large block slides on a flat table. All surfaces are frictionless. Both blocks are initially at rest. Gravity acts downward and $g = 9.81$ m/s^2. Assume neither block rotates.

a. (25 pts) Find the speed v_A of the small block just as it leaves the large block.

   Hint: it might help to consider both blocks together as a system of particles. Is energy conserved between $t_{initial}$ and t_{final} for this system? Is linear momentum conserved? (Is linear momentum conserved in one direction but not the other?)

Extra credit (mystery blocks).

```
function mysteryblocks

[t,y] = ode45(@f,[0,2*pi],[0 0.5 0 0]);
figure(1);
plot(t,y(:,1),'k-',t,y(:,3),'k--');
figure(2);
plot(t,y(:,2),'k-',t,y(:,4),'k--');

function dy=f(t,y)

m1=1;
m2=1;
k=2;

y1 = y(1,1);
y2 = y(2,1);
y3 = y(3,1);
y4 = y(4,1);

dy(1,1) = y2;
dy(2,1) = (k/m1)*(y3-y1);
dy(3,1) = y4;
dy(4,1) = (k/m2)*(y1-y3);
```

The MATLAB code given above integrates the equations of motion for a simple mechanical system consisting of masses and springs.

a. (3 pts) Draw a picture of this system, labeling all variables.

b. (4 pts) Write the equations of motion for this system, and describe the initial conditions.

c. (3 pts) Sketch the two plots produced by `mysteryblocks`. They need not be exact, but must indicate any known quantities (such as maxima/minima and initial/final values).