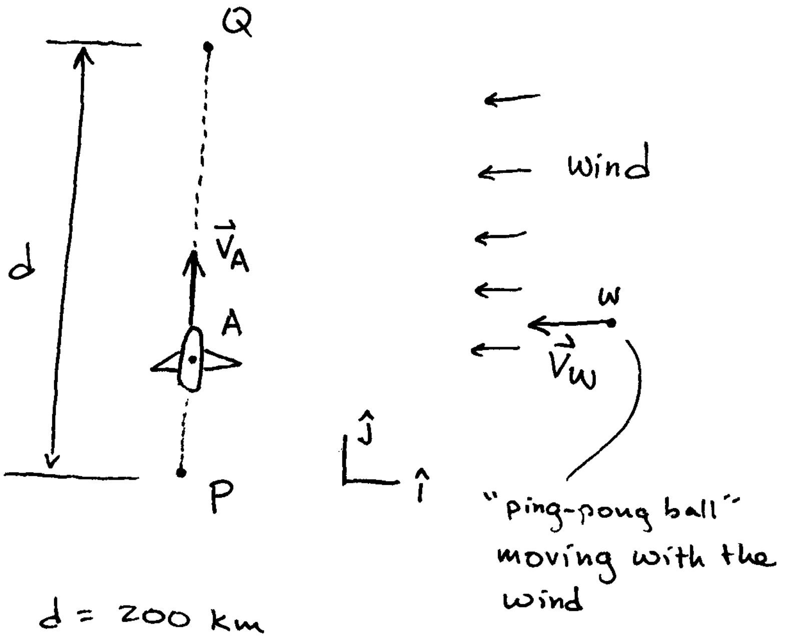
2/1/07

Bretl

AE252 (Spring 2007)

Example: Relative Motion (text problem 13.168)



The pilot wants to fly directly from P to Q. The plane's airspeed is V_0. The windspeed is V_W. What direction should the pilot point the plane, and how long does it take to reach Q?

$d = 200$ km

$V_0 = 290$ km/h

$V_W = 50$ km/h

First, let's answer some easier questions ...

1. No wind $\rightsquigarrow$ $\vec{V}_W = 0$. So $\vec{V}_A = V_0 \hat{j}$.

   The plane points in the $\hat{j}$ direction and reaches Q in $t = d/V_0 = 41.4$ minutes.

2. Tailwind $\rightsquigarrow$ $\vec{V}_W = V_W \hat{j}$. So $\vec{V}_A = (V_0 + V_W)\hat{j}$.

   Again, point in $\hat{j}$ direction and $t = \dfrac{d}{V_0 + V_W} = 35.3$ minutes.

3. Headwind $\rightsquigarrow$ $\vec{V}_W = -V_W \hat{j}$. So $\vec{V}_A = (V_0 - V_W)\hat{j}$.

   Once again, point in $\hat{j}$ direction and $t = \dfrac{d}{V_0 - V_W} = 50$ minutes

These answers are fairly intuitive.

Now we consider the question as posed in 13.168.

$$\vec{V}_W = -V_W \hat{\imath}$$

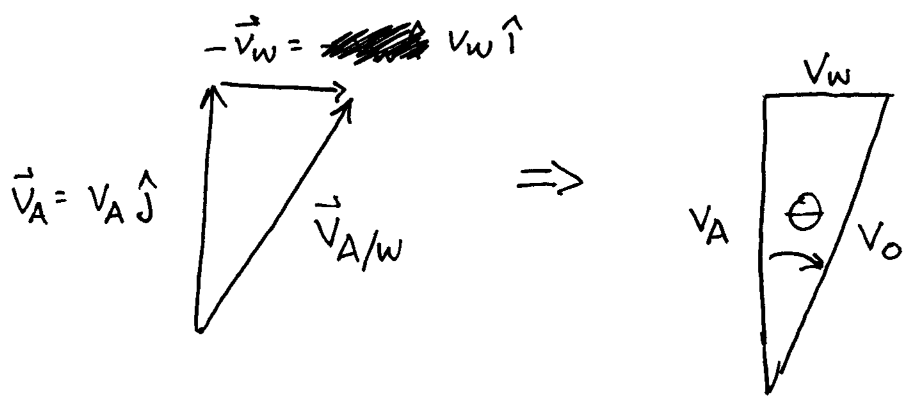
We want to achieve an actual airplane velocity of

$$\vec{V}_A = V_A \hat{\jmath}$$

so we move directly from P to Q, but we don't know V_A. Let $\vec{V}_{A/W}$ be the velocity of the plane relative to the wind (relative to the "ping-pong ball."). Then we know by definition

$$\vec{V}_{A/W} = \vec{V}_A - \vec{V}_W .$$

The "airspeed" is the plane's speed relative to the wind, so we also know $V_0 = \|\vec{V}_{A/W}\|$.

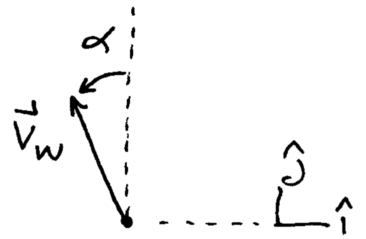


Consequently, $\sin\theta = \dfrac{V_W}{V_0} \Rightarrow \theta = \sin^{-1}\left(\dfrac{V_W}{V_0}\right) = 9.9°$.

So the plane should always try to fly in a direction 9.9° east of north — it will actually be flying due north b/c of the wind!

$$\cos\theta = \frac{V_A}{V_0} \Rightarrow V_A = V_0\cos\theta \Rightarrow t = \frac{d}{V_A} = \frac{d}{V_0\cos\theta} = 42 \text{ minutes.}$$

Let's answer one more question. What if



$$\vec{V}_W = V_W(-\sin\alpha\,\hat{i} + \cos\alpha\,\hat{j}),$$

so the wind at an angle α west of north. When ...

wind helps!

$\alpha = 0 \longrightarrow$ tailwind

$\alpha = 90° \longrightarrow$ crosswind (the text problem)

$\alpha = 180° \longrightarrow$ headwind

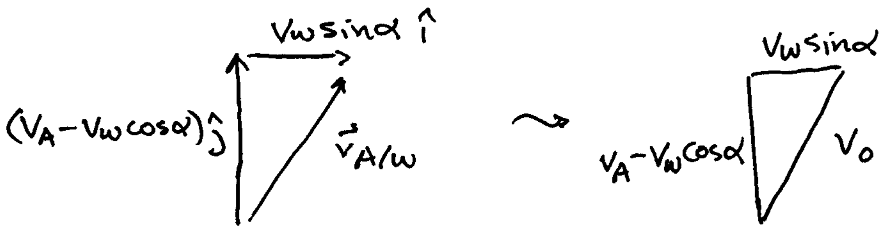
wind hurts!

At what angle α does the wind start to slow the plane down? Recall that with no wind,

$$\boxed{V_A = V_0}.$$

We want to find α such that $V_A = V_0$!

$$\vec{V}_{A/W} = \vec{V}_A - \vec{V}_W = V_W \sin\alpha\,\hat{i} + (V_A - V_W\cos\alpha)\hat{j}$$



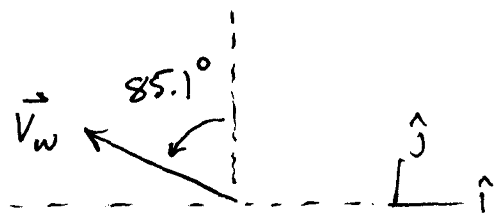
$$V_0^2 = V_A^2 - 2V_AV_W\cos\alpha + V_W^2\cos^2\alpha + V_W^2\sin^2\alpha$$

When $V_A = V_0$ ... $\quad 0 = -2V_0V_W\cos\alpha + V_W^2\underbrace{(\sin^2\alpha + \cos^2\alpha)}_{=1}$

$$\Rightarrow \quad 2V_0V_W\cos\alpha = V_W^2$$

$$\Rightarrow \quad \alpha = \cos^{-1}\left(\frac{V_W}{2V_0}\right) = 85.1°$$

So in this example, the wind neither helps nor hurts when it is blowing 85.1° west of north.



Some questions to consider on your own:

1. Is this problem symmetric? (I.e., what if the wind was blowing to the right and not the left? What part of your answer is the same, and what is different?)

2. Is it always fastest to fly directly from P to Q? (That is, to fly straight so $\vec{V}_A = V_A \hat{j}$ in global coordinates.) Or is it better to fly a curvilinear path? How might you prove one or the other?