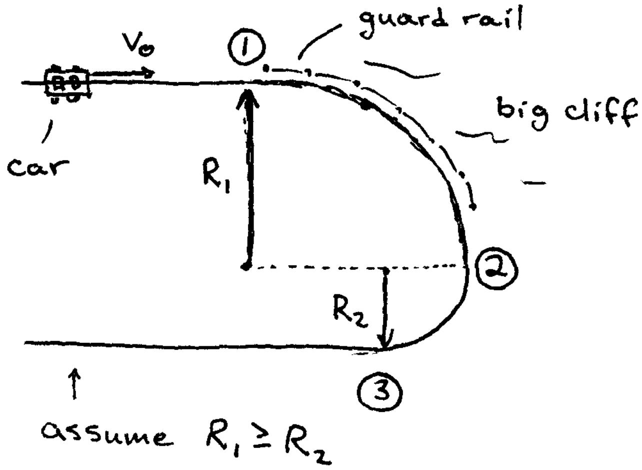
1/30/07

Bretl

AE 252 (Spring 2007)

Example: path coordinates



A car drives at constant speed v_0 toward a decreasing-radius turn. Assume the tires maintain rolling contact up to an acceleration (magnitude) of K. Clearly, if K is small and v_0 is large, then the car is in trouble.

We are asked to place a sign indicating the maximum recommended speed v_{max}. Assume the driver decelerates at a constant rate a_0 when he/she sees the sign, until his/her speed is v_{max}.

— Where should we place the sign?

(a) Nowhere. (No sign needed.)

(b) So it can be seen from ②.

(c) So it can be seen from ①.

(d) So it can be seen <u>before</u> ①.

— What speed should it recommend?

In any case we know $\vec{a} = \dot{v}\hat{e}_t + \frac{v^2}{r_c}\hat{e}_n$

so to guarantee no slip we must have

$$\|\vec{a}\| = \sqrt{\dot{v}^2 + \left(\frac{v^2}{r_c}\right)^2} \leq K$$

at all times.

(a) Is no sign required? So $\dot{v} = 0$ / $v = v_0$ always.

Before 1 and after 3, $\|\vec{a}\| = 0 \leq K$.

1 → 2 $\quad \|\vec{a}\| = v_0^2/R_1$

2 → 3 $\quad \|\vec{a}\| = v_0^2/R_2$

So we must have $\boxed{\frac{v_0^2}{R_1} \leq K \quad \text{and} \quad \frac{v_0^2}{R_2} \leq K}$.

(b) Is a sign at 2 required?

Before 1 and after 3, $\|\vec{a}\| = 0 \leq K$.

1 → 2 $\quad \dot{v} = 0 \Rightarrow \|\vec{a}\| = v_0^2/R_1$. So $\boxed{\frac{v_0^2}{R_1} \leq K}$.

2 → 3 $\quad$ From (a), we only need a sign if $\boxed{v_0^2/R_2 > K}$. Then, $\dot{v} = -a_0$.

$$\Rightarrow \|\vec{a}\| = \sqrt{a_0^2 + \left(v_0^2/R_2\right)^2} > \frac{v_0^2}{R_2} > K$$

at point 2. So a sign at 2 is $\boxed{\text{never a good idea!}}$ I.e., it never helps.

(c) Is a sign at 1 required?

(As usual, before 1 and after 3 $\|\vec{a}\| = 0 \le K$.)

1 → 2 $\quad \dot{v} = -a_0 \Rightarrow \|\vec{a}\| = \sqrt{a_0^2 + (v^2/R_1)^2}$

Note we must have $\boxed{V_0^2/R_1 \le K}$. Also note

that since v is maximum at 1, where $v = V_0$,

we must have $\sqrt{a_0^2 + (V_0^2/R_1)^2} \le K$

$$\Rightarrow \quad a_0^2 \le K^2 - (V_0^2/R_1)^2$$

$$\Rightarrow \quad a_0 \le \sqrt{K^2 - (V_0^2/R_1)^2}$$

2 → 3 $\quad$ We have $\boxed{V_0^2/R_2 > K}$. We want to decelerate

from 1 → 2 at least so that $V_2^2/R_2 = K$

$$\Rightarrow \quad V_2 = \sqrt{KR_2} \quad \rightsquigarrow \quad \boxed{V_{max} = \sqrt{KR_2}}$$

Then, $\|\vec{a}\| = \frac{V_{max}^2}{R_2} \le K$ between 2 and 3.

How fast must we decelerate?

$$-a_0 = \frac{dv}{dt} = \frac{dv}{ds}\frac{ds}{dt} = v\frac{dv}{ds}$$

$$\Rightarrow \int_{V_1}^{V_2} v\,dv = -a_0 \int_{s_1}^{s_2} ds \Rightarrow \frac{V_2^2 - V_1^2}{2} = -a_0(s_2 - s_1)$$

$V_1 = V_0$ $\qquad V_2 \le \sqrt{KR_2}$ $\qquad s_2 - s_1 = \frac{\pi R_1}{2}$

$$\Rightarrow \quad V_2^2 = V_0^2 - \cancel{2}a_0\left(\frac{\pi R_1}{\cancel{2}}\right) \le KR_2$$

$$\Rightarrow \quad a_0 \ge (V_0^2 - KR_2)/\pi R_1$$

→ So we must have $\boxed{\sqrt{K^2 - (V_0^2/R_1)^2} \ge a_0 \ge \frac{V_0^2 - KR_2}{\pi R_1}}$

(d) Is a sign before 1 required?

From (a)–(c) we must have:

$$\frac{v_0^2}{R_2} > K$$

must slow down before second curve or we crash

AND

$$\frac{v_0^2 - KR_2}{\pi R_1} > a_0$$ ← can't slow down fast enough before second curve

AND/OR

$$\sqrt{K^2 - \left(\frac{v_0^2}{R_1}\right)^2} < a_0$$ ← if we slow down at rate a_0 in the first curve, we crash

AND/OR

$$\frac{v_0^2}{R_1} > K$$ ← must slow down before first curve or we crash

Strategy — find the maximum v_0 that satisfies all conditions, then post sign far enough before 1 so the car slows (at rate a_0) to v_0 by the time it reaches the first turn.

COMMENTS:

- In practice, a_0 is not fixed. What is a good choice? How might we encourage a particular a_0? What if a_0 varies with time? Does this help? Can we move through the turn faster?
- Why is it a good idea to keep our solution algebraic?