Brett
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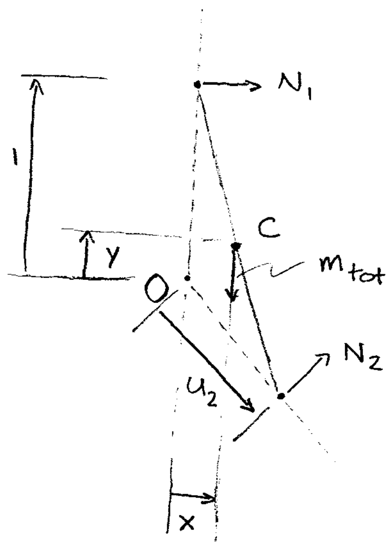
# Note about text problem 18.67 ← HW #11, problem 4




$m_{tot}\, g = (m_{BAR} + 2 m_{SLIDER})g$

Many people solved this problem using the expression

$$\sum \vec{M}_c = \frac{{}^{\mathcal{F}}d\vec{H}_c}{dt} = I_c \alpha_B \hat{k}$$

(that is, taking moments about the center of mass).

It is also possible to use the expression

$$\sum \vec{M}_O = \frac{{}^{\mathcal{F}}d\vec{H}_O}{dt}$$

for the fixed point O as defined in my picture.

However, it is <u>not</u> correct to say

$$\frac{{}^{\mathcal{F}}d\vec{H}_O}{dt} = I_O \alpha_B \hat{k} = (I_c + m_{tot}(x^2+y^2))\alpha \hat{k}$$

because O is <u>not</u> fixed in B (that is, the bar is not rotating about O).

Instead, we must start with

$$\vec{H}_O = \vec{H}_c + \vec{r}_c \times m_{tot} {}^{\mathcal{F}}\vec{V}_c \qquad \text{(derived in class)}$$

$$\Rightarrow \frac{{}^{\mathcal{F}}d\vec{H}_O}{dt} = \frac{{}^{\mathcal{F}}d\vec{H}_c}{dt} + \cancel{{}^{\mathcal{F}}\vec{V}_c \times m_{tot} {}^{\mathcal{F}}\vec{V}_c}^{\,0} + \vec{r}_c \times m_{tot} {}^{\mathcal{F}}\vec{a}_c$$

$$= I_c \alpha_B \hat{k} + (x\hat{i} + y\hat{j}) \times m_{tot}(\ddot{x}\hat{i} + \ddot{y}\hat{j})$$

$$= (I_c \alpha_B + m_{tot}(x\ddot{y} - y\ddot{x}))\hat{k}$$

Next we compute

$$\Sigma \vec{M}_O = \left( -u_1 N_1 + u_2 N_2 - m_{tot}\, g\, x \right) \hat{k}$$

(this step was pretty easy, one reason you might choose this particular approach).

So we find

$$\boxed{-u_1 N_1 + u_2 N_2 - m_{tot}\, g\, x = I_c \alpha_B + m_{tot}\left( x\ddot{y} - y\ddot{x} \right)}$$

This is a correct expression of the rotational dynamics. (Note you can relate $\ddot{x}, \ddot{y}$ to $\alpha_B$ as usual.) Compare this to what you would have gotten using $\frac{{}^{\mathcal{F}}d\vec{H}_O}{dt} = I_O \alpha_B \hat{k}$ —

$$-u_1 N_1 + u_2 N_2 - m_{tot}\, g\, x = \underbrace{\left( I_c + m_{tot}\left( x^2 + y^2 \right) \right) \alpha_B}_{\text{incorrect!}}$$