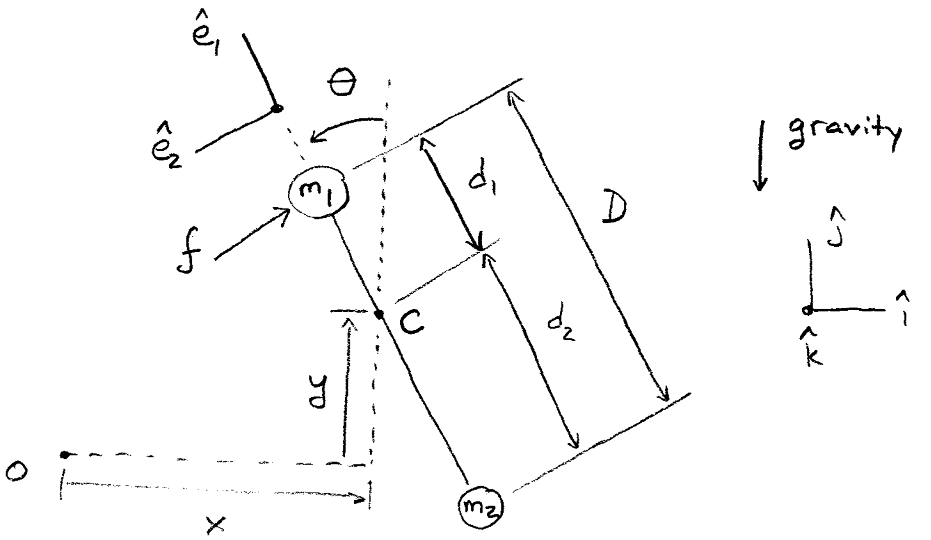
AE252 (Spring 2007)

Example: falling astronaut with thruster



(1) Find the center of mass. We will use $\sum m_i \vec{r}_{i/c} = 0$.

$$m_1 d_1 \hat{e}_1 - m_2 d_2 \hat{e}_1 = 0$$

$$\Rightarrow \quad m_1 d_1 - m_2 (D - d_1) = 0$$

$$\Rightarrow \quad (m_1 + m_2) d_1 = m_2 D$$

$$\Rightarrow \quad \boxed{d_1 = \left(\frac{m_2}{m_1+m_2}\right) D}, \quad \boxed{d_2 = \left(\frac{m_1}{m_1+m_2}\right) D}.$$

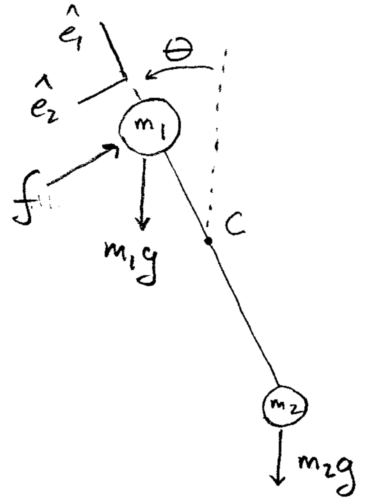
(2) Find mass moment of inertia, about the center of mass.

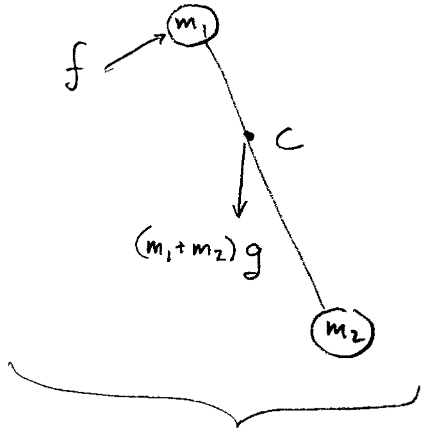
$$I_c = \sum m_i r_i^2 = m_1 d_1^2 + m_2 d_2^2$$

$$= m_1 \frac{m_2^2}{(m_1+m_2)^2} D^2 + m_2 \frac{m_1^2}{(m_1+m_2)^2} D^2$$

$$= D^2 \frac{m_1 m_2}{(m_1+m_2)^2} (m_2 + m_1) \quad \Rightarrow \quad \boxed{I_c = \left(\frac{m_1 m_2}{m_1+m_2}\right) D^2}$$

(3) Draw a free-body diagram.





We may as well draw the combined force of gravity acting at the center of mass. Why?

Compute the moment exerted by gravity about C.M. –

$$\sum \vec{M}_{c,\,grav} = (d_1 \hat{e}_1) \times (-m_1 g \hat{j}) + (-d_2 \hat{e}_1) \times (-m_2 g \hat{j})$$

$$= (\hat{e}_1 \times \hat{j}) \left[-m_1 g \left(D \frac{m_2}{m_1+m_2} \right) + m_2 g \left(D \frac{m_1}{m_1+m_2} \right) \right]$$

$$= (\hat{e}_1 \times \hat{j}) \cdot 0$$

$$= 0.$$

So gravity exerts no moment about the center of mass. (Can you think of a situation where this would *not* be true?)

4 Find acceleration and angular acceleration.

$$\vec{r}_c = x\hat{\imath} + y\hat{\jmath} \Rightarrow {}^{\mathcal{F}}\vec{a}_c = \ddot{x}\hat{\imath} + \ddot{y}\hat{\jmath}.$$

$${}^{\mathcal{F}}\vec{\omega}^{B} = \dot{\theta}\hat{k} \Rightarrow \alpha_B = \ddot{\theta} \longleftarrow \text{planar motion!}$$

5 Find forces and moments.

$$\Sigma\vec{F} = -f\hat{e}_2 - (m_1+m_2)g\hat{\jmath} = f\cos\theta\,\hat{\imath} + (f\sin\theta - (m_1+m_2)g)\hat{\jmath}$$

$$\Sigma\vec{M} = (d_1\hat{e}_1)\times(-f\hat{e}_2) = -fd_1\hat{e}_3 = -fD\left(\frac{m_2}{m_1+m_2}\right)\hat{k}$$

6 Translational motion: $\Sigma\vec{F} = m_c\,{}^{\mathcal{F}}\vec{a}_c$

$$f\cos\theta\,\hat{\imath} + (f\sin\theta - (m_1+m_2)g)\hat{\jmath} = (m_1+m_2)\left[\ddot{x}\hat{\imath} + \ddot{y}\hat{\jmath}\right]$$

$$\Rightarrow \boxed{\begin{aligned}\ddot{x} &= \frac{f\cos\theta}{m_1+m_2}\\ \ddot{y} &= \frac{f\sin\theta}{m_1+m_2} - g\end{aligned}} \longleftarrow \text{translational EOMs}$$

7 Rotational motion: $\Sigma\vec{M}_c = I_c\alpha_B\hat{k}$

$$-fD\left(\frac{\cancel{m_2}}{\cancel{m_1+m_2}}\right)\hat{k} = \left(\frac{m_1\cancel{m_2}}{\cancel{m_1+m_2}}\right)D^{\cancel{2}}\ddot{\theta}\,\hat{k}$$

$$\Rightarrow \boxed{\ddot{\theta} = -\frac{f}{m_1 D}} \longleftarrow \text{rotational EOM}$$

At this point, we could integrate to find the position and angle of the "astronaut" over time (given the input $f(t)$).