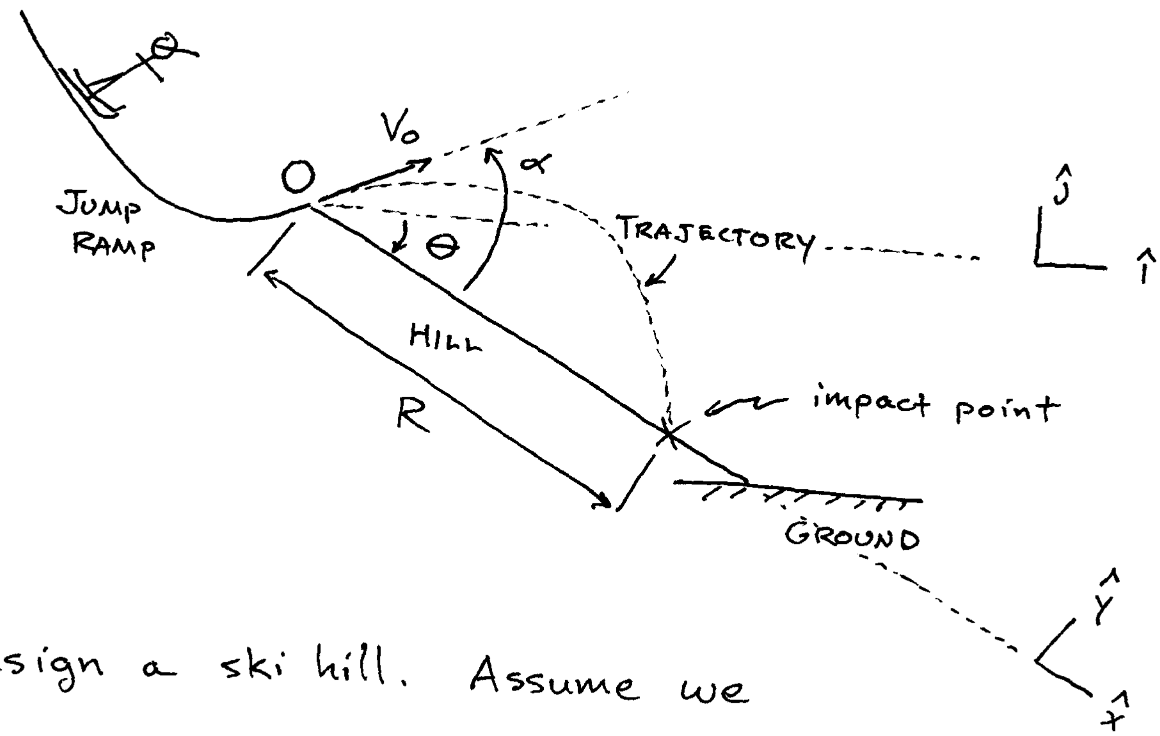
AE252 (Spring 2007) 1/18/07

Bretl

Example: planar motion, constant acceleration, rotated coordinate system



Design a ski hill. Assume we are given the maximum initial velocity v_0, the angle θ of the hill, and the length R of the hill (before a jumper hits the "ground"). Choose the ramp angle α so the jumper lands at R. (Assume constant acceleration $\vec{a} = -g\hat{\jmath}$.)

A choice that makes things easier: use a coordinate system $\hat{x}, \hat{y}$ aligned with the hill.

Notice that $\hat{j} = -\sin\theta\,\hat{x} + \cos\theta\,\hat{y}$.

So $\vec{a} = -g\hat{j} = g\sin\theta\,\hat{x} - g\cos\theta\,\hat{y}$.

$$\vec{v} = (v_0\cos\alpha + g\sin\theta\, t)\,\hat{x} + (v_0\sin\alpha - g\cos\theta\, t)\,\hat{y}$$

$$\vec{r} = \left(v_0\cos\alpha\, t + g\sin\theta\,\frac{t^2}{2}\right)\hat{x} + \left(v_0\sin\alpha\, t - g\cos\theta\,\frac{t^2}{2}\right)\hat{y}$$

WHEN does the skier hit? When $\vec{r}\cdot\hat{y} = 0$.

$$v_0\sin\alpha\, t - g\cos\theta\,\frac{t^2}{2} = 0$$

$$t = 0 \quad \text{or} \quad t = \frac{2v_0\sin\alpha}{g\cos\theta}$$

WHERE does the skier hit? At $\vec{r}\cdot\hat{x} = R$.

$$R = v_0\cos\alpha\left(\frac{2v_0\sin\alpha}{g\cos\theta}\right) + \frac{g\sin\theta}{2}\left(\frac{4v_0^2\sin^2\alpha}{g^2\cos^2\theta}\right)$$

$$\Rightarrow \quad R = \left(\frac{2v_0^2}{g}\right)\left(\frac{\sin\alpha\cos\alpha}{\cos\theta} + \frac{\sin^2\alpha\,\sin\theta}{\cos^2\theta}\right)$$

At this point, we might want to solve numerically for α, given (for example)

$R = 15$ meters $g = 9.8\ \frac{\text{meters}}{\text{second}^2}$

$\theta = 30^\circ$ $v_0 = 10\ \frac{\text{meters}}{\text{second}}$

We find $\alpha = 34°$ or $\alpha = 86°$. Did you <u>expect</u> to have a choice? Since you do, which angle might you choose? What if you wanted the jumper's velocity <u>normal</u> to the hill at impact to be minimized? Is it possible to make this velocity <u>zero</u> by changing θ? What if you wanted to <u>maximize</u> R?