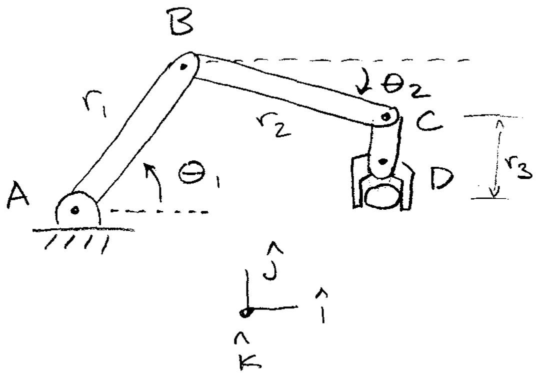
Bretl

AE 252 (Spring 2007)

3/29/07

Example: robot arm (text 17.110)



We want CD to remain vertical while ${}^{\mathcal{F}}\vec{v}_D = 1\,\hat{i}$ (m/s) and ${}^{\mathcal{F}}\vec{a}_D = 0$.

Assume $r_1 = 0.3$ m, $r_2 = 0.3$ m, $r_3 = 0.17$ m, $\theta_1 = 50°$, $\theta_2 = 15°$.

Find the required angular velocities and angular accelerations.

VELOCITIES:

$$\vec{v}_B = \vec{v}_A + \vec{\omega}^{AB} \times \vec{r}_{B/A}$$

$$= 0 + \dot{\theta}_1 \hat{k} \times r_1 (\cos\theta_1 \hat{i} + \sin\theta_1 \hat{j})$$

$$= r_1 \dot{\theta}_1 (-\sin\theta_1 \hat{i} + \cos\theta_1 \hat{j})$$

$$\vec{v}_C = \vec{v}_B + \vec{\omega}^{BC} \times \vec{r}_{C/B}$$

$$= \vec{v}_B + (-\dot{\theta}_2 \hat{k}) \times r_2 (\cos\theta_2 \hat{i} - \sin\theta_2 \hat{j})$$

$$= \vec{v}_B + r_2 \dot{\theta}_2 (-\sin\theta_2 \hat{i} - \cos\theta_2 \hat{j})$$

$$= (-r_1 \dot{\theta}_1 \sin\theta_1 - r_2 \dot{\theta}_2 \sin\theta_2)\hat{i} + (r_1 \dot{\theta}_1 \cos\theta_1 - r_2 \dot{\theta}_2 \cos\theta_2)\hat{j}$$

We are given $\vec{v}_D = 1\,\hat{i}$. Note $\vec{v}_D = \vec{v}_C + \cancelto{0}{\vec{\omega}^{CD}} \times \vec{r}_{D/C} = \vec{v}_C$.

$$\Rightarrow \quad \left.\begin{aligned} 1 &= -r_1 \dot{\theta}_1 \sin\theta_1 - r_2 \dot{\theta}_2 \sin\theta_2 \\ 0 &= r_1 \dot{\theta}_1 \cos\theta_1 - r_2 \dot{\theta}_2 \cos\theta_2 \end{aligned}\right\}$$

Solve to find

$\dot{\theta}_1 = -3.6$ rad/s

$\dot{\theta}_2 = -2.4$ rad/s

$$\Rightarrow \quad \boxed{\vec{\omega}^{AB} = -3.6\,\hat{k}, \quad \vec{\omega}^{BC} = 2.4\,\hat{k}}$$

ACCELERATIONS:

$$\vec{a}_B = \vec{a}_A + \vec{\alpha}^{AB} \times \vec{r}_{B/A} + \vec{\omega}^{AB} \times (\vec{\omega}^{AB} \times \vec{r}_{B/A})$$

$$= 0 + \alpha_{AB}\hat{k} \times r_1(\cos\theta_1\hat{i} + \sin\theta_1\hat{j}) + \omega_{AB}\hat{k} \times (\omega_{AB}\hat{k} \times r_1(\cos\theta_1\hat{i} + \sin\theta_1\hat{j}))$$

$$= r_1\alpha_{AB}(-\sin\theta_1\hat{i} + \cos\theta_1\hat{j}) + r_1\omega_{AB}^2(\hat{k} \times (-\sin\theta_1\hat{i} + \cos\theta_1\hat{j}))$$

$$= r_1\alpha_{AB}(-\sin\theta_1\hat{i} + \cos\theta_1\hat{j}) + r_1\omega_{AB}^2(-\cos\theta_1\hat{i} - \sin\theta_1\hat{j})$$

$$\vec{a}_C = \vec{a}_B + \vec{\alpha}^{BC} \times \vec{r}_{C/B} + \vec{\omega}^{BC} \times (\vec{\omega}^{BC} \times \vec{r}_{C/B})$$

$$= \vec{a}_B + (-\alpha_{BC}\hat{k}) \times r_2(\cos\theta_2\hat{i} - \sin\theta_2\hat{j}) + \omega_{BC}\hat{k} \times (\omega_{BC}\hat{k} \times r_2(\cos\theta_2\hat{i} - \sin\theta_2\hat{j}))$$

$$= \vec{a}_B - r_2\alpha_{BC}(\sin\theta_2\hat{i} + \cos\theta_2\hat{j}) + r_2\omega_{BC}^2\ \hat{k} \times (\cos\theta_2\hat{j} + \sin\theta_2\hat{i})$$

$$= \vec{a}_B - r_2\alpha_{BC}(\sin\theta_2\hat{i} + \cos\theta_2\hat{j}) + r_2\omega_{BC}^2(-\cos\theta_2\hat{i} + \sin\theta_2\hat{j})$$

$$= (-r_1\alpha_{AB}\sin\theta_1 - r_1\omega_{AB}^2\cos\theta_1 - r_2\alpha_{BC}\sin\theta_2 - r_2\omega_{BC}^2\cos\theta_2)\hat{i}$$
$$+ (r_1\alpha_{AB}\cos\theta_1 - r_1\omega_{AB}^2\sin\theta_1 - r_2\alpha_{BC}\cos\theta_2 + r_2\omega_{BC}^2\sin\theta_2)\hat{j}$$

NOTE $\vec{a}_D = \vec{a}_C$, and we are given $\vec{a}_D = 0$.

$$\Rightarrow \begin{cases} 0 = -r_1\alpha_{AB}\sin\theta_1 - r_1\omega_{AB}^2\cos\theta_1 - r_2\alpha_{BC}\sin\theta_2 - r_2\omega_{BC}^2\cos\theta_2 \\ 0 = r_1\alpha_{AB}\cos\theta_1 - r_1\omega_{AB}^2\sin\theta_1 - r_2\alpha_{BC}\cos\theta_2 + r_2\omega_{BC}^2\sin\theta_2 \end{cases}$$

given $\omega_{AB} = \dot{\theta}_1$, $\omega_{BC} = \dot{\theta}_2$ from before, solve to find $\alpha_{AB} = \ddot{\theta}_1$ and $\alpha_{BC} = \ddot{\theta}_2$ $\leadsto$ $\ddot{\theta}_1 = -12.1$ rad/s^2

$\ddot{\theta}_2 = -16.5$ rad/s^2

$$\Rightarrow \boxed{\vec{\alpha}^{AB} = -12.1\,\hat{k},\quad \vec{\alpha}^{BC} = 16.5\,\hat{k}}$$