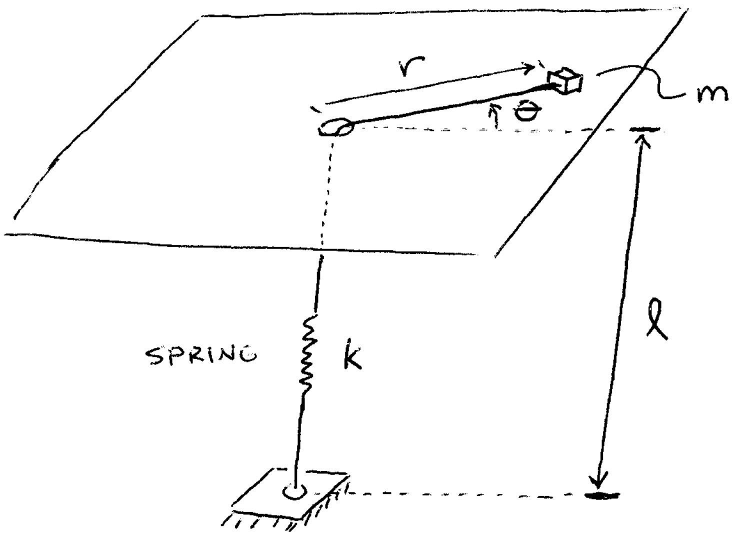
2/8/07

AE252 (Spring 2007)

Bretl

Example: newton's laws (numerical integration, part 2)



A small block of mass $m = 1$ kg is set spinning on a table. It is connected by an elastic string to a fixed base. The unstretched length of the string is ℓ, so r is equal to the amount of stretch. Assume the block slides without friction (so it will never drop through the hole).

Let $k = 65$ N/m and define the initial conditions

$$r_0 = 1 \text{ m}, \quad \dot{r}_0 = 0 \text{ m/s}, \quad \theta_0 = 0^\circ, \quad \dot{\theta}_0 = 5 \text{ rad/s}.$$

(a) Plot r as a function of time to determine when the block is closest to the hole ($t = t_{MIN}$) and how close it gets, $r(t_{MIN})$.

(b) Plot the trajectory of the block from $t=0$ to $t = t_{MIN}$ in Cartesian coordinates.

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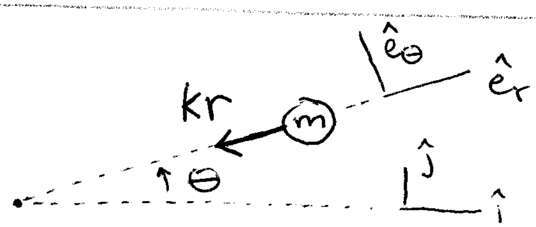
① Free-body diagram.



② Coordinate axes.

The spring force acts in the $-\hat{e}_r$ direction $\Rightarrow$ polar coordinates.

③ Acceleration.

$$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\,\hat{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\,\hat{e}_\theta$$

④ Forces.

$$\Sigma \vec{F} = -kr\,\hat{e}_r$$

⑤ Newton's law.

$$m(\ddot{r} - r\dot{\theta}^2) = -kr$$

$$m(r\ddot{\theta} + 2\dot{r}\dot{\theta}) = 0$$

⑥ Equations of motion.

$$\boxed{\begin{aligned} \ddot{r} &= r\dot{\theta}^2 - \frac{k}{m} r \\ \ddot{\theta} &= -2\dot{r}\dot{\theta}/r \end{aligned}}$$

⑦ Solve for quantities of interest.

We will integrate the equations of motion numerically using MATLAB.

(i) Write equations of motion in MATLAB format.

Let $y_1 = r$, $y_2 = \dot{r}$, $y_3 = \theta$, $y_4 = \dot{\theta}$.

$$\Rightarrow \boxed{\begin{aligned} \frac{dy_1}{dt} &= y_2 \\ \frac{dy_2}{dt} &= y_1 y_4^2 - \frac{k}{m} y_1 \\ \frac{dy_3}{dt} &= y_4 \\ \frac{dy_4}{dt} &= -2 y_2 y_4 / y_1 \end{aligned}}$$

and initial conditions $y_0 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 5 \end{bmatrix}$.

(ii) For part (b), express the trajectory in Cartesian coordinates:

$$\vec{r} = r\hat{e}_r = r\cos\theta\,\hat{\imath} + r\sin\theta\,\hat{\jmath} = x\hat{\imath} + y\hat{\jmath}$$

$$\Rightarrow \boxed{\begin{aligned} x &= r\cos\theta \\ y &= r\sin\theta \end{aligned}}$$

For the rest, see the attached file blockspring.m.

We find $t_{MIN} = 0.195\,s$ and $r(t_{MIN}) \approx 0.62\,m$.